

AD-A245 605



NAVAL POSTGRADUATE SCHOOL
Monterey, California



DTIC
ELECTE
FEB 07, 1992
S, D

T H E S I S

**Dynamic Stability of Positively Buoyant
Submersibles: Vertical Plane Solutions**

by

Brian D. McKinley

December 1991

Thesis Advisor

Fotis A. Papoulias

Approved for public release; distribution is unlimited.

92-03145



Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

| | | | | | |
|---|-------|--|--|-------------------------|----------------------------------|
| 1a. REPORT SECURITY CLASSIFICATION Unclassified | | | 1b. RESTRICTIVE MARKINGS | | |
| 2a. SECURITY CLASSIFICATION AUTHORITY | | | 3. DISTRIBUTION AVAILABILITY OF REPORT Approved for public release; distribution is unlimited. | | |
| 2b. DECLASSIFICATION/DOWNGRADING SCHEDULE | | | | | |
| 4. PERFORMING ORGANIZATION REPORT NUMBER(S) | | | 5. MONITORING ORGANIZATION REPORT NUMBER(S) | | |
| 6a. NAME OF PERFORMING ORGANIZATION Naval Postgraduate School | | 6b. OFFICE SYMBOL (If Applicable) ME | 7a. NAME OF MONITORING ORGANIZATION Naval Postgraduate School | | |
| 6c. ADDRESS (city, state, and ZIP code) Monterey, CA 93943-5000 | | | 7b. ADDRESS (city, state, and ZIP code) Monterey, CA 93943-5000 | | |
| 8a. NAME OF FUNDING/SPONSORING ORGANIZATION | | 8b. OFFICE SYMBOL (If Applicable) | 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER | | |
| 8c. ADDRESS (city, state, and ZIP code) | | | 10. SOURCE OF FUNDING NUMBERS | | |
| PROGRAM ELEMENT NO. | | PROJECT NO. | TASK NO. | WORK UNIT ACCESSION NO. | |
| 11. Title (Include Security Classification) Dynamic Stability of Positively Buoyant Submersibles: Vertical Plane Solutions | | | | | |
| 12. PERSONAL AUTHOR(S) McKinley, Brian D. | | | | | |
| 13a. TYPE OF REPORT Master's Thesis | | 13b. TIME COVERED FROM Jul 91 TO Dec91 | 14. DATE OF REPORT (year, month, day) 1991 December | | 15. PAGE COUNT 103 |
| 16. SUPPLEMENTARY NOTATION The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government. | | | | | |
| 17. COSATI CODES | | | 18. SUBJECT TERMS (continue on reverse if necessary and identify by block number) | | |
| FIELD | GROUP | SUBGROUP | Dynamic Stability of Positively Buoyant Submersibles, Positively Buoyant Submersibles, Submersibles, Excess Buoyancy | | |
| | | | | | |
| | | | | | |
| 19. ABSTRACT (Continue on reverse if necessary and identify by block number) | | | | | |
| <p>This thesis analyzes the dynamic stability of positively buoyant submersibles. Six degree-of-freedom equations of motion are used to compute steady state behavior with motion restricted to the vertical plane. Steady state solutions are analyzed for various conditions of buoyancy including changes in (1) the amount of excess buoyancy, (2) the location of the center of buoyancy, (3) the location of the center of gravity, as well as (4) the deflection of bow and stern planes. The equations of motion are then linearized around these steady state solutions to predict dynamic response in the vertical plane. The stability of each solution is then determined by eigen value analysis. The study then expands the analysis to include all six degrees of freedom (i.e., include stability analysis in the horizontal plane). Finally, numerical integration methods are used to verify the results.</p> | | | | | |
| 20. DISTRIBUTION AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED RPT. DTIC USERS | | | 21. ABSTRACT SECURITY CLASSIFICATION Unclassified | | |
| 22a. NAME OF RESPONSIBLE INDIVIDUAL Professor F.A. Papoulias | | | 22b. TELEPHONE (Include Area Code) (408) 646-3381 | | 22c. OFFICE SYMBOL Code ME/Pa |

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted

SECURITY CLASSIFICATION OF THIS PAGE

All other editions are obsolete

Unclassified

Approved for public release; distribution is unlimited.

**Dynamic Stability of Positively Buoyant
Submersibles: Vertical Plane Solutions**

by

**Brian D. McKinley
Lieutenant, United States Navy
B.S., Virginia Polytechnic Institute and State University, 1979**

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

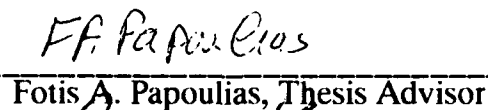
NAVAL POSTGRADUATE SCHOOL

December 1991

Author:


Brian D. McKinley

Approved by:


Fotis A. Papoulas, Thesis Advisor


Anthony J. Healey, Chairman
Department of Mechanical Engineering

ABSTRACT

This thesis analyzes the dynamic stability of positively buoyant submersibles. Six degree-of-freedom equations of motion are used to compute steady state behavior with motion restricted to the vertical plane. Steady state solutions are analyzed for various conditions of buoyancy including changes in (1) the amount of excess buoyancy, (2) the location of the center of buoyancy, (3) the location of the center of gravity, as well as (4) the deflection of bow and stern planes. The equations of motion are then linearized around these steady state solutions to predict dynamic response in the vertical plane. The stability of each solution is then determined by eigen value analysis. The study then expands the analysis to include all six degrees of freedom (i.e., include stability analysis in the horizontal plane). Finally, numerical integration methods are used to verify the results.



| | |
|--------------------|--|
| Accession For | |
| NTIS GRA&I | <input checked="checked" type="checkbox"/> |
| DTIC TAB | <input type="checkbox"/> |
| Unannounced | <input type="checkbox"/> |
| Justification | |
| By | |
| Distribution/ | |
| Availability Codes | |
| Dist | Avail and/or Special |
| A-1 | |

TABLE OF CONTENTS

| | |
|--|----------|
| I. INTRODUCTION..... | 1 |
| A. PROBLEM STATEMENT..... | 1 |
| B. EQUATIONS OF MOTION | 2 |
| 1. Dynamic Variables..... | 2 |
| 2. Mass Distribution Parameters | 3 |
| 3. Remaining Parameters | 3 |
| II. SYSTEM SOLUTIONS IN THE VERTICAL PLANE..... | 6 |
| A. GENERAL..... | 6 |
| B. CONDITIONS..... | 7 |
| 1. Defining Additional Terms | 7 |
| a. Excess Buoyancy, δB | 7 |
| b. Longitudinal Center of Buoyancy, x_{GB} | 8 |
| c. Vertical Center of Buoyancy, z_{GB} | 8 |
| 2. Assumed Conditions | 8 |
| a. Lateral Centers of Gravity, y_G , and Buoyancy, y_B | 8 |
| b. Propeller Speed, n (revolutions per minute)..... | 8 |
| c. Propeller Coefficients, K_{prop} and N_{prop} | 8 |
| C. REVISING THE EQUATIONS OF MOTION..... | 8 |
| D. ADDITIONAL CONDITIONS | 10 |
| E. VERTICAL PLANE EQUATIONS OF MOTION..... | 11 |
| F. COMPUTER PROGRAM DEVELOPMENT | 11 |
| G. STEADY STATE RESULTS..... | 16 |

| | |
|---|-----------|
| III. DYNAMIC STABILITY..... | 20 |
| A. GENERAL..... | 20 |
| B. RESTRICTING MOTION TO THE VERTICAL PLANE..... | 20 |
| C. LINEARIZED VERTICAL PLANE EQUATIONS OF MOTION..... | 20 |
| D. VERTICAL PLANE COMPUTER PROGRAM DEVELOPMENT | 27 |
| E. VERTICAL PLANE DYNAMIC RESULTS..... | 27 |
| F. LINEARIZATION OF FULL EQUATIONS OF MOTION | 30 |
| G. COMPUTER PROGRAM DEVELOPMENT | 43 |
| H. DYNAMIC STABILITY SOLUTIONS..... | 43 |
| IV. SIMULATIONS AND DISCUSSION OF RESULTS..... | 47 |
| A. DYNAMIC STABILITY ANALYSIS RESULTS..... | 47 |
| 1. Variations in Longitudinal Center of Gravity (x_{GB})..... | 47 |
| 2. Variations in the Amount of Excess Buoyancy (δB)..... | 52 |
| 3. Variations in Vertical Center of Gravity (z_{GB}) | 57 |
| 4. Variations in the Longitudinal Center of Buoyancy (x_B)..... | 59 |
| 5. Variations in Bow Planes Deflection Angle (δ_b)..... | 62 |
| B. SIMULATIONS USING NUMERICAL INTEGRATION METHODS..... | 64 |
| VI. CONCLUSIONS AND RECOMMENDATIONS. | 72 |
| APPENDIX A. SIX DEGREE OF FREEDOM EQUATIONS | |
| OF MOTION..... | 73 |
| APPENDIX B. ROTATION SEQUENCE AND EULER ANGLE | |
| RATES | 76 |
| APPENDIX C. STEADY STATE COMPUTER PROGRAM..... | 78 |
| APPENDIX D. LINEARIZED DYNAMIC STABILITY COMPUTER | |
| PROGRAM..... | 83 |

| | | |
|--|-----------|---|
| LIST OF REFERENCES..... | 91 | . |
| BIBLIOGRAPHY..... | 91 | |
| INITIAL DISTRIBUTION LIST | 92 | . |

LIST OF FIGURES

| | |
|--|----|
| 1. Schematic Indicating Positive Directions of Axes, Angles, Velocities, Forces, and Moments | 5 |
| 2. Steady State Vertical Plane Solutions for Surge Velocity (u) | 17 |
| 3. Steady State Vertical Plane Solutions for Heave Velocity (w) | 18 |
| 4. Steady State Vertical Plane Solutions for Pitch Angle (θ) | 19 |
| 5. Stable Vertical Plane Solutions for Surge Velocity | 28 |
| 6. Stable Vertical Plane Solutions for Heave Velocity | 29 |
| 7. Stable Vertical Plane Solutions for Pitch Angle (θ) | 29 |
| 8. Degree of Stability in the Vertical Plane | 30 |
| 9. Degree of Stability in the Horizontal Plane | 45 |
| 10. Degree of Stability in Both (Horizontal and Vertical) Planes | 45 |
| 11. Degree of Stability as a Function of Surge Velocity (u) for a Neutrally Buoyant Submersible | 46 |
| 12. Stable Surge Velocity (u) Solutions for Variations in x_{GB} | 48 |
| 13. Stable Heave Velocity (w) Solutions for Variations in x_{GB} | 49 |
| 14. Stable Angle of Pitch (θ) Solutions for Variations in x_{GB} | 50 |
| 15. Degree of Stability for Variations in x_{GB} | 51 |
| 16. Stable Surge Velocity (u) Solutions for Variations in δB | 53 |
| 17. Stable Heave Velocity (w) Solutions for Variations in δB | 54 |
| 18. Stable Angle of Pitch (θ) Solutions for Variations in δB | 55 |
| 19. Degree of Stability for Variations in δB | 56 |
| 20. Stable Surge Velocity (u) Solutions for Variations in z_{GB} | 57 |
| 21. Stable Heave Velocity (w) Solutions for Variations in z_{GB} | 58 |

| | |
|--|----|
| 22. Stable Angle of Pitch (θ) Solutions for Variations in z_{GB} | 58 |
| 23. Degree of Stability for Variations in z_{GB} | 59 |
| 24. Stable Surge Velocity (u) Solutions for Variations in x_B | 60 |
| 25. Stable Heave Velocity (w) Solutions for Variations in x_B | 60 |
| 26. Stable Angle of Pitch (θ) Solutions for Variations in x_B | 61 |
| 27. Degree of Stability for Variations in x_B | 61 |
| 28. Stable Surge Velocity (u) Solutions for a Non-zero Bow Plane Angle ($\delta_b = -20$ degrees) | 62 |
| 29. Stable Heave Velocity (w) Solutions for a Non-zero Bow Plane Angle ($\delta_b = -20$ degrees) | 63 |
| 30. Stable Pitch Angle (θ)Solutions for a Non-zero bow Plane Angle ($\delta_b = -20$ degrees) | 63 |
| 31. Degree of Stability for a Non-zero Bow Plane Angle ($\delta_b = -20$ degrees) | 64 |
| 32. Numerical Integration Solution for Angle of Pitch (θ) when Center of Gravity is Forward ($x_{GB} = +1\%$)..... | 65 |
| 33. Numerical Integration Solution for Angle of Pitch (θ) when Center of Gravity is Aft ($x_{GB} = -1\%$) | 66 |
| 34. Numerical Integration Solution for Angle of Pitch (θ) when $x_{GB} = -1\%$ and Initial Roll Angle (ϕ_0) is 1 degree | 67 |
| 35. Numerical Integration Solution for Angle of Roll (ϕ) when $x_{GB} = -1\%$ and Initial Roll Angle (ϕ_0) is 1 degree | 67 |
| 36. Numerical Integration Solution for Yaw Velocity (r) when $x_{GB} = -1\%$ and Initial Roll Angle (ϕ_0) is 1 degree | 68 |

| | |
|---|----|
| 37. Numerical Integration Solution for Heave Velocity (z) when $x_{GB} = -1\%$ and Initial Roll Angle (ϕ_0) is 1 degree | 68 |
| 38. Numerical Integration Solution for Angle of Pitch (θ) when $x_{GB} = -1\%$ and a Persistent Roll Disturbance is Added | 70 |
| 39. Numerical Integration Solution for Angle of Roll (ϕ) when $x_{GB} = -1\%$ and a Large Persistent Roll Disturbance is Added..... | 70 |
| 40. Numerical Integration Solution for Angle of Roll (ϕ) when $x_{GB} = -1\%$ and a Small Persistent Roll Disturbance is Added..... | 71 |
| B1. Unit Sphere Development of Euler Angles | 77 |

I. INTRODUCTION

A. PROBLEM STATEMENT

Controlling emergency ascent situations on submersible vehicles such as dive plane jam recovery is of concern to the U.S. Navy. In order to control such situations, one must first be able to predict the dynamic response of positively buoyant submersibles.

Dynamic response equations of motion describe the maneuvering characteristics of submersible vehicles for six degrees of freedom. These equations assume constant coefficients for hydrodynamic forces and moments approximated by zero frequency added mass and damping terms plus the quadratic terms for drag forces. The constant coefficients vary for each vehicle and are dependent on such things as vehicle body shape, location and magnitude of vehicle weight, location and magnitude of vehicle buoyancy, position of bow and stern planes, position of rudder, vehicle speed, vehicle mass characteristics, vehicle hydrodynamic coefficients, propeller rpm and control surface inputs. This thesis uses the equations of motion and hydrodynamic coefficients for a submerged Mark IX Swimmer Delivery Vehicle (SDV) developed by Smith, Crane, and Summey [Reference 1:11-16,21-31] to forecast the dynamic stability of a submersible in a positively buoyant condition.

This study begins by using the six equations of motion to compute the steady state behavior of a submersible vehicle with motion restricted to the vertical plane. The steady state solutions in the vertical plane are calculated for various conditions of buoyancy including changes in the amount of excess buoyancy, the location of the center of buoyancy, the location of the center of gravity, as well as the deflection of bow and stern planes. The SDV's equations of motion are then linearized around these steady state solutions to predict dynamic response motion in the vertical plane for the various conditions

of buoyancy. Several solutions are computed and the stability of each solution is determined by eigen value analysis. The thesis then expands the analysis to include all six degrees of freedom (i.e. include stability analysis in the horizontal plane). Finally, numerical integration methods are used to verify the results.

B. EQUATIONS OF MOTION

The six degree of freedom equations of motion for the submersible vehicle shown in Appendix A were taken from Smith, Crane, and Summey [Reference 1:11-16]. Differentiation with respect to time is denoted by a dot over the variable; e.g. $\dot{u} = \frac{du}{dt}$. These equations are referenced to a right-hand orthogonal axis system fixed in the body (vehicle) as shown in Figure 1. Since these equations are in reference to a body fixed axis system, the Euler angles of pitch (θ), roll (ϕ), and yaw (ψ) are used to specify orientation with respect to the inertial reference system. The rotation sequence for ϕ , θ and ψ , and the Euler angle rates for $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ shown in Appendix B were taken from: Smith, Crane, and Summey [Reference 1:18-20]. Major variables and parameters as defined by Smith, Crane, and Summey [Reference 1:7-10] are given below :

1. Dynamic Variables

- | | |
|-----------|--|
| u, v, w | - Linear velocity components of vehicle with respect to origin of body axes system relative to fluid. |
| p, q, r | - Angular velocity components of vehicle with respect to body axes system relative to inertial reference system. |
| X, Y, Z | - Hydrodynamic force components along body axes. |
| K, M, N | - Hydrodynamic moment components along body axes. |

2. Mass Distribution Parameters

- m - Mass of the flooded vehicle, including the mass of the entrained fluid.
- W - Weight of the flooded vehicle, including the weight of the entrained fluid ($= g m$; where g is the acceleration of gravity).
- V - Displacement volume of the vehicle.
- B - Buoyancy force acting on the vehicle ($= \rho g V$). This is independent of the inertial mass distribution of the submersible vehicle, including whether or not it is flooded.
- x_G, y_G, z_G - Coordinates of the CG (center of gravity) in the body axis system (Figure 1). These will depend on the mass distribution of the vehicle, including the mass of the entrained fluid.
- x_B, y_B, z_B - Coordinates of the CB (center of buoyancy) in the body axis system (Figure 1). These are independent of the mass distribution system, but may vary with the addition or removal of external appendages.
- I_x, I_y, I_z - Moments of inertia about the body system axes, including the entrained fluid.
- I_{xy}, I_{xz}, I_{yz} - Products of inertia about the body system axes, including the entrained fluid.

3. Remaining Parameters

- ρ - Mass density of fluid medium
- l - Reference length used to nondimensionalize the hydrodynamic coefficients.

- | | |
|------------------------------------|---|
| $b(x), h(x)$ | - Width and height of vehicle in its xy and xz planes, respectively, at location x measured in the body axes system (Figure 1). These quantities are required in the integrals defining the crossflow forces and moments in the equations of motion, and are tabulated within the Steady State Computer Program (Appendix C). |
| $x_{\text{nose}}, x_{\text{tail}}$ | - Sternplane, bowplane and rudder deflection angles in radians (Figure 1). |

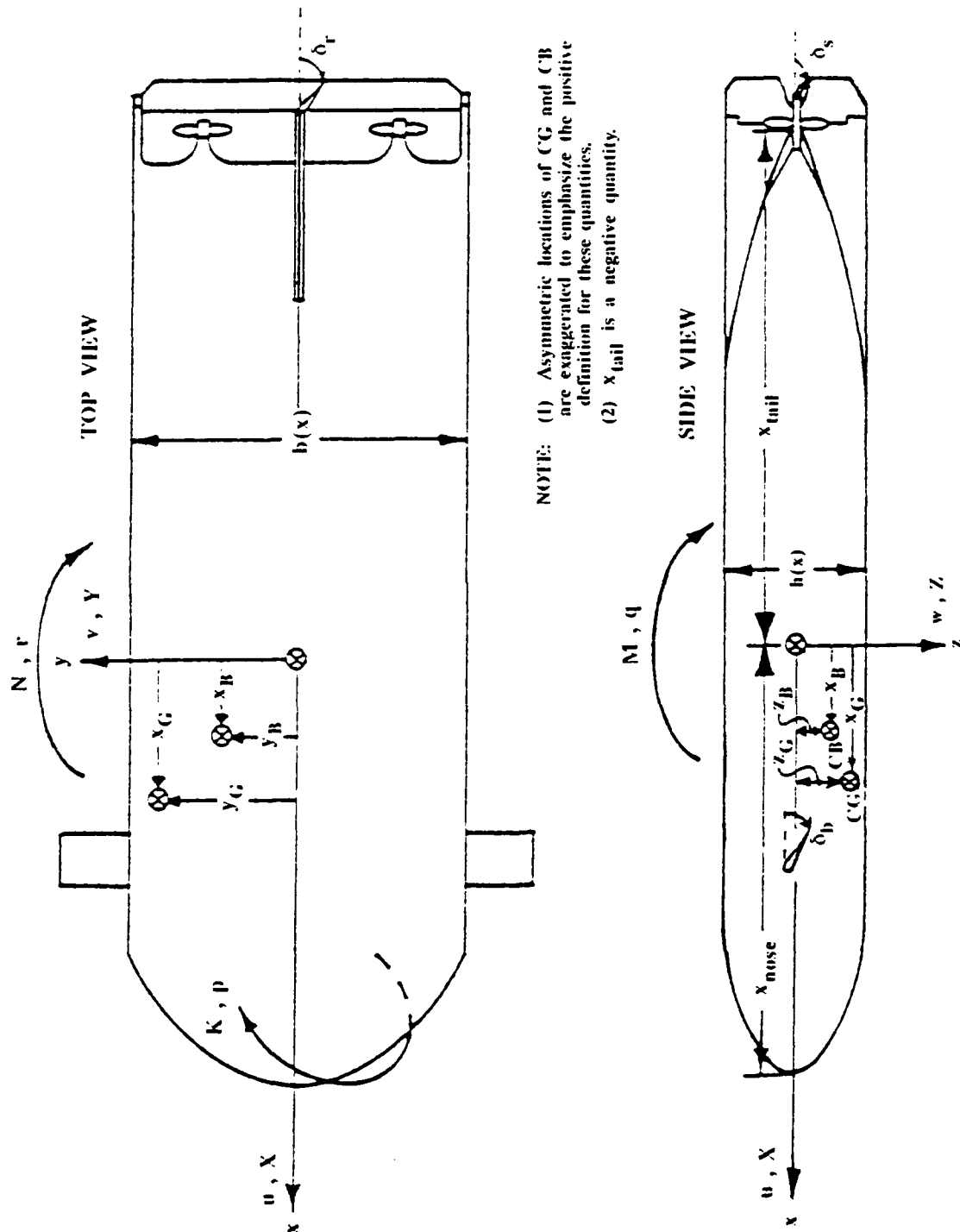


Figure 1. Schematic Indicating Positive Directions of Axes, Angles, Velocities, Forces, and Moments

II. SYSTEM SOLUTIONS IN THE VERTICAL PLANE

A. GENERAL

In the steady state condition, the submersible will have reached constant linear and angular velocities. Therefore, the body fixed linear accelerations (\ddot{u} , \ddot{v} , \ddot{w}) and the body fixed angular accelerations (\ddot{p} , \ddot{q} , \ddot{r}) will be zero. Similarly, the vehicle will have reached a constant angle of pitch (θ) making its derivatives ($\dot{\theta}$) equal to zero. Since this analysis is restricted to steady state solutions in the vertical plane, the angle of roll (ϕ) and its derivative ($\dot{\phi}$) will be zero (in Chapter IV, the case where the angle of roll (ϕ) is 180 degrees will be discussed during the numerical integration analysis). The angle of yaw (ψ) and its derivative ($\dot{\psi}$) will likewise be zero due to the vertical plane restriction. It should be noted that had this analysis not been restricted to the vertical plane, steady state yaw (ψ) would not necessarily be zero, thereby allowing the angular velocities (p , q , r) to be non-zero. However, since this analysis was restricted to the cases where $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$, p , q and r are all zero, the equations of motion for six degrees of freedom for the steady state condition can be reduced to:

- Longitudinal (Surge) Equation of Motion:

$$(W - B) \sin \theta = \left[X_{vv} v^2 + X_{ww} w^2 + X_{v\delta r} uv \delta_r + uw \left(X_{w\delta s} \delta_s + X_{w\delta b} \delta_b \right) \right] + u^2 \left[X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2 + X_{\delta r \delta r} \delta_r^2 \right] + u^2 X_{prop}$$

- Lateral (Sway) Equation of Motion:

$$-(W-B) \cos \theta \sin \phi = \left[Y_v uv + Y_{vw} vw + Y_{\delta r} u^2 \delta_r \right] - \int_{x_{tail}}^{x_{nose}} \left[C_{Dy} h(x) (v)^2 + C_{Dz} b(x) (w)^2 \right] \frac{v}{U_{cf}(x)} dx$$

- Normal (Heave) Equation of Motion

$$-(W-B) \cos \theta \cos \phi = [Z_w uw + Z_{vv} v^2 + u^2 (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b)] - \int_{x_{tail}}^{x_{nose}} [C_{Dy} h(x) (v)^2 + C_{Dz} b(x) (w)^2] \frac{w}{U_{cf}(x)} dx$$
- Roll Equation of Motion:

$$-(y_G W - y_B B) \cos \theta \cos \phi + (z_G W - z_B B) \cos \theta \sin \phi = [K_v uv + K_{vw} vw] + u^2 K_{prop}$$
- Pitch Equation of Motion:

$$(x_G W - x_B B) \cos \theta \cos \phi + (z_G W - z_B B) \sin \theta = [M_w uw + M_{vv} v^2 + u^2 (M_{\delta s} \delta_s + M_{\delta b} \delta_b)] + \int_{x_{tail}}^{x_{nose}} [C_{Dy} h(x) (v)^2 + C_{Dz} b(x) (w)^2] \frac{w}{U_{cf}(x)} x dx$$
- Yaw Equation of Motion:

$$-(x_G W - x_B B) \cos \theta \sin \phi + (y_G W - y_B B) \sin \theta = [N_v uv + N_{vw} vw + N_{\delta r} u^2 \delta_r] + u^2 N_{prop} - \int_{x_{tail}}^{x_{nose}} [C_{Dy} h(x) (v)^2 + C_{Dz} b(x) (w)^2] \frac{v}{U_{cf}(x)} x dx$$

B. CONDITIONS

1. Defining Additional Terms

a. *Excess Buoyancy, δB*

Excess buoyancy is defined as $\delta B = B - W$ where B is the submersible's total buoyancy and W is the submersible's total weight.

b. Longitudinal Center of Buoyancy, x_{GB}

The longitudinal center of buoyancy is defined as $x_{GB} = x_G - x_B$ where x_G is the longitudinal center of gravity with respect to the body fixed axis and x_B is the longitudinal center of buoyancy with respect to the body fixed axis.

c. Vertical Center of Buoyancy, z_{GB}

The vertical center of buoyancy is defined as $z_{GB} = z_G - z_B$ where z_G is the longitudinal center of gravity with respect to the body fixed axis and z_B is the longitudinal center of buoyancy with respect to the body fixed axis (z_{GB} is assumed to be positive).

2. Assumed Conditions

a. Lateral Centers of Gravity, y_G , and Buoyancy, y_B

The lateral center of gravity and the center of buoyancy are assumed to be on the same centerline plane ($y_G = y_B = 0$).

b. Propeller Speed, n (revolutions per minute)

The propeller speed is assumed to be zero ($n = 0$).

c. Propeller Coefficients, K_{prop} and N_{prop}

From Smith, Crane, and Summey [Reference 1:30], the propeller coefficients are zero ($K_{prop} = N_{prop} = 0$).

C. REVISING THE EQUATIONS OF MOTION

Using the term for vertical center of buoyancy (z_{GB}), the expression ($z_G W - z_B B$) may be written as ($z_{GB} W - z_B \delta B$). Similarly, using the term for longitudinal center of buoyancy (x_{GB}), the expression ($x_G W - x_B B$) may be written as ($x_{GB} W - x_B \delta B$). Also, the term $u^2 X_{prop}$ may be written as:

$$u^2 X_{prop} = u^2 C_{D0} (\eta^2 - 1) = u^2 C_{D0} \left[\left(\frac{u_{commanded}}{u_{actual}} \right)^2 - 1 \right] = C_{D0} A^2 n^2 - C_{D0} u^2$$

where A is a constant.

Since the shaft speed (n) is zero, the expression may be further reduced to $u^2 X_{prop} = -C_{D0} u^2$. Substituting these expressions plus the term for excess buoyancy (δB) and the assumed conditions revises the equations of motion for the six degree of freedom system as follows:

- Longitudinal (Surge) Equation of Motion:

$$-\delta B \sin \theta = X_{vv} v^2 + X_{ww} w^2 + X_{v\delta r} uv \delta_r + uw \left(X_{w\delta s} \delta_s + X_{w\delta b} \delta_b \right) + u^2 \left(X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2 + X_{\delta r \delta r} \delta_r^2 \right) - C_{D0} u^2$$

- Lateral (Sway) Equation of Motion:

$$\delta B \cos \theta \sin \phi = Y_v uv + Y_{vw} vw + Y_{\delta r} u^2 \delta_r - \int_{x_{tail}}^{x_{nose}} \left[C_{Dy} h(x) v^2 + C_{Dz} b(x) w^2 \right] \frac{v}{U_{cf}(x)} dx$$

- Normal (Heave) Equation of Motion:

$$\delta B \cos \theta \cos \phi = Z_w uw + Z_{vv} v^2 + u^2 \left(Z_{\delta s} \delta_s + Z_{\delta b} \delta_b \right) - \int_{x_{tail}}^{x_{nose}} \left[C_{Dy} h(x) v^2 + C_{Dz} b(x) w^2 \right] \frac{w}{U_{cf}(x)} dx$$

- Roll Equation of Motion:

$$(z_{GB} W - z_B \delta B) \cos \theta \sin \phi = K_v uv + K_{vw} vw$$

- Pitch Equation of Motion:

$$(x_{GB} W - x_B \delta B) \cos \theta \cos \phi + (z_{GB} W - z_B \delta B) \sin \theta = M_w uw + M_{vv} v^2 + u^2 \left(M_{\delta s} \delta_s + M_{\delta b} \delta_b \right) + \int_{x_{tail}}^{x_{nose}} \left[C_{Dy} h(x) v^2 + C_{Dz} b(x) w^2 \right] \frac{w}{U_{cf}(x)} x dx$$

- Yaw Equation of Motion:

$$(-x_{GB} W + x_B \delta B) \cos \theta \sin \phi = [N_v uv + N_{vw} vw + N_{\delta r} u^2 \delta_r] - \int_{x_{tail}}^{x_{nose}} [C_{Dy} h(x) v^2 + C_{Dz} b(x) w^2] \frac{v}{U_{cf}(x)} x dx$$

These six equations only have five unknowns: u, v, w, θ , and ϕ . Therefore, two of these equations must be dependent and additional conditions are required in order to make the number of equations equal the number of unknowns.

D. ADDITIONAL CONDITIONS

The next condition to be applied to the vehicle is that the rudder will remain centerlined, that is $\delta r = 0$. Since the solutions of interest are those in which the vehicle remains within the vertical plane, it can be further specified that the linear velocity in the transverse direction (v) equals zero. Recalling that the angle of roll (ϕ) has been previously assumed to equal zero, the trigonometric functions of ϕ can be identified as $\sin \phi$ equals zero and $\cos \phi$ equals unity. Substituting these quantities back into the equations of motion, three of the six equations of motion (sway, roll, and yaw) yield trivial solutions. In addition, the cross-flow velocity term (U_{cf}) for the heave and pitch equations can be reduced to:

$$U_{cf}(x) = [(v + xr)^2 + (w - xq)^2]^{0.5} = [w^2]^{0.5} = |w|$$

since v, r , and q are zero. Furthermore, since C_{Dz} is constant, it can be taken outside the integral. Therefore, the three remaining equations can be written as:

- Longitudinal (Surge) Equation of Motion:

$$-\delta B \sin \theta = X_{ww} w^2 + uw (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) + u^2 (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2) - C_{D0} u^2$$

- Normal (Heave) Equation of Motion:

$$\delta B \cos \theta = Z_w u w + u^2 (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) - C_{Dz} \int_{x_{tail}}^{x_{nose}} b(x) \frac{w^3}{|w|} dx$$

- Pitch Equation of Motion:

$$(x_{GB} W - x_B \delta B) \cos \theta + (z_{GB} W - z_B \delta B) \sin \theta =$$

$$M_w u w + u^2 (M_{\delta s} \delta_s + M_{\delta b} \delta_b) + C_{Dz} \int_{x_{tail}}^{x_{nose}} b(x) \frac{w^3}{|w|} x dx$$

E. VERTICAL PLANE EQUATIONS OF MOTION

By defining the terms A_w as the $\int_{x_{tail}}^{x_{nose}} b(x) dx$ and x_A as $\frac{1}{A_w} \int_{x_{tail}}^{x_{nose}} b(x) x dx$, the three

remaining equations defining motion in the vertical plane can be written as:

- Longitudinal (Surge) Equation of Motion:

$$- \delta B \sin \theta = X_{ww} w^2 + u w (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b)$$

$$+ u^2 (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2) - C_{D0} u^2$$

- Normal (Heave) Equation of Motion:

$$\delta B \cos \theta = Z_w u w + u^2 (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) - C_{Dz} w |w| A_w$$

- Pitch Equation of Motion:

$$(x_{GB} W - x_B \delta B) \cos \theta + (z_{GB} W - z_B \delta B) \sin \theta =$$

$$M_w u w + u^2 (M_{\delta s} \delta_s + M_{\delta b} \delta_b) + C_{Dz} w |w| x_A A_w$$

F. COMPUTER PROGRAM DEVELOPMENT

Taking these three equations which describe motion in the vertical plane, solving the first two for sine θ and cosine θ , respectively; and dividing all three through by u^2 yields:

- Longitudinal (Surge) Equation of Motion:

$$\frac{\sin \theta}{u^2} = -\frac{1}{\delta B} \left[X_{ww} \left(\frac{w}{u}\right)^2 + \left(\frac{w}{u}\right) (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) + (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2) - C_{D0} \right]$$

- Normal (Heave) Equation of Motion:

$$\frac{\cos \theta}{u^2} = \frac{1}{\delta B} \left[Z_w \frac{w}{u} + (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) - C_{Dz} \frac{w |w|}{u^2} A_w \right]$$

- Pitch Equation of Motion:

$$\begin{aligned} & (x_{GB} W - x_B \delta B) \frac{\cos \theta}{u^2} + (z_{GB} W - z_B \delta B) \frac{\sin \theta}{u^2} = \\ & M_w \frac{w}{u} + (M_{\delta s} \delta_s + M_{\delta b} \delta_b) + C_{Dz} \frac{w |w|}{u^2} x_A A_w \end{aligned}$$

Now, defining the quantity $\frac{w}{u}$ as w' , and substituting w' back into the three equations:

if w is positive:

- Longitudinal (Surge) Equation of Motion:

$$\frac{\sin \theta}{u^2} = -\frac{1}{\delta B} \left[X_{ww} (w')^2 + (w') (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) + (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2) - C_{D0} \right]$$

- Normal (Heave) Equation of Motion:

$$\frac{\cos \theta}{u^2} = \frac{1}{\delta B} \left[Z_w (w') + (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) - C_{Dz} (w')^2 A_w \right]$$

- Pitch Equation of Motion:

$$\begin{aligned} & (x_{GB} W - x_B \delta B) \frac{\cos \theta}{u^2} + (z_{GB} W - z_B \delta B) \frac{\sin \theta}{u^2} = \\ & M_w (w') + (M_{\delta s} \delta_s + M_{\delta b} \delta_b) + C_{Dz} (w')^2 x_A A_w \end{aligned}$$

however, if w is negative:

- Longitudinal (Surge) Equation of Motion:

$$\frac{\sin \theta}{u^2} = -\frac{1}{\delta B} \left[X_{ww} (w')^2 + (w') (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) + (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2) - C_{D0} \right]$$

- Normal (Heave) Equation of Motion:

$$\frac{\cos \theta}{u^2} = \frac{1}{\delta B} [Z_w (w') + (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) + C_{Dz} (w')^2 A_w]$$

- Pitch Equation of Motion:

$$(x_{GB} W - x_B \delta B) \frac{\cos \theta}{u^2} + (z_{GB} W - z_B \delta B) \frac{\sin \theta}{u^2} =$$

$$M_w (w') + (M_{\delta s} \delta_s + M_{\delta b} \delta_b) - C_{Dz} (w')^2 x_A A_w$$

Substituting the equations for $\frac{\sin \theta}{u^2}$ and $\frac{\cos \theta}{u^2}$ into the pitch equation yields the following expressions:

if w is positive:

$$(x_{GB} W - x_B \delta B) \frac{1}{\delta B} [Z_w (w') + (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) - C_{Dz} (w')^2 A_w]$$

$$- (z_{GB} W - z_B \delta B) \frac{1}{\delta B} [X_{ww} (w')^2 + (w') (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) + (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2) - C_{D0}] =$$

$$M_w (w') + (M_{\delta s} \delta_s + M_{\delta b} \delta_b) + C_{Dz} (w')^2 x_A A_w$$

and if w is negative:

$$(x_{GB} W - x_B \delta B) \frac{1}{\delta B} [Z_w (w') + (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) + C_{Dz} (w')^2 A_w]$$

$$- (z_{GB} W - z_B \delta B) \frac{1}{\delta B} [X_{ww} (w')^2 + (w') (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) + (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2) - C_{D0}] =$$

$$M_w (w') + (M_{\delta s} \delta_s + M_{\delta b} \delta_b) - C_{Dz} (w')^2 x_A A_w$$

Rearranging these two equations to get them into the form: $A(w')^2 + B(w') + C = 0$, the expressions become:

if w is positive:

$$\begin{aligned}
 & \left[C_{Dz} x_A A_w + (z_{GB} W - z_B \delta B) \frac{1}{\delta B} X_{ww} \right] (w')^2 \\
 & + \left(x_{GB} W - x_B \delta B \right) \frac{1}{\delta B} C_{Dz} A_w \\
 & + \left[M_w + (z_{GB} W - z_B \delta B) \frac{1}{\delta B} (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) \right. \\
 & \quad \left. - (x_{GB} W - x_B \delta B) \frac{1}{\delta B} Z_w \right] (w') \\
 & + \left[(z_{GB} W - z_B \delta B) \frac{1}{\delta B} (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2 - C_{D0}) \right. \\
 & \quad \left. + (M_{\delta s} \delta_s + M_{\delta b} \delta_b) - (x_{GB} W - x_B \delta B) \frac{1}{\delta B} (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) \right] = 0
 \end{aligned}$$

and if w is negative:

$$\begin{aligned}
 & \left[-C_{Dz} x_A A_w + (z_{GB} W - z_B \delta B) \frac{1}{\delta B} X_{ww} \right] (w')^2 \\
 & - \left(x_{GB} W - x_B \delta B \right) \frac{1}{\delta B} C_{Dz} A_w \\
 & + \left[M_w + (z_{GB} W - z_B \delta B) \frac{1}{\delta B} (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) \right. \\
 & \quad \left. - (x_{GB} W - x_B \delta B) \frac{1}{\delta B} Z_w \right] (w') \\
 & + \left[(z_{GB} W - z_B \delta B) \frac{1}{\delta B} (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2 - C_{D0}) \right. \\
 & \quad \left. + (M_{\delta s} \delta_s + M_{\delta b} \delta_b) - (x_{GB} W - x_B \delta B) \frac{1}{\delta B} (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) \right] = 0
 \end{aligned}$$

These quadratic expressions were then solved using the equation:

$$w' = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where:

$$\begin{aligned}
 B &= \delta B M_w + (z_{GB} W - z_B \delta B) (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) \\
 &\quad - (x_{GB} W - x_B \delta B) Z_w \\
 C &= (z_{GB} W - z_B \delta B) (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2 - C_{D0}) \\
 &\quad + \delta B (M_{\delta s} \delta_s + M_{\delta b} \delta_b) - (x_{GB} W - x_B \delta B) (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b)
 \end{aligned}$$

if w is positive:

$$A = \delta B (C_{Dz} x_A A_w) + (z_{GB} W - z_B \delta B) X_{ww} + (x_{GB} W - x_B \delta B) C_{Dz} A_w;$$

and if w is negative:

$$A = -\delta B(C_{DZ} x_A A_w) + (z_{GB} W - z_B \delta B) X_{ww} - (x_{GB} W - x_B \delta B) C_{DZ} A_w$$

The value of θ was determined using the computed values of w' and the equation for tangent θ :

$$\tan \theta = \frac{\sin \theta}{\frac{\cos \theta}{u^2}}$$

However, the value of $\frac{\cos \theta}{u^2}$ varied depending on the value of w' , which lead to two possible solutions:

Equation for $\tan \theta$ if w' is Positive:

$$\tan \theta = \frac{-X_{ww}(w')^2 - (w')(X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) - (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2) + C_{D0}}{Z_w(w') + (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) - C_{DZ} A_w(w')w'}$$

Equation for $\tan \theta$ if w' is Negative:

$$\tan \theta = \frac{-X_{ww}(w')^2 - (w')(X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) - (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2) + C_{D0}}{Z_w(w') + (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b) + C_{DZ} A_w(w')w'}$$

In either case, the value of u^2 was computed using the expression derived from the surge equation of motion:

$$u^2 = \frac{\delta B \sin \theta}{-X_{ww}(w')^2 - (w')(X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) - (X_{\delta s \delta s} \delta_s^2 + X_{\delta b \delta b} \delta_b^2) + C_{D0}}$$

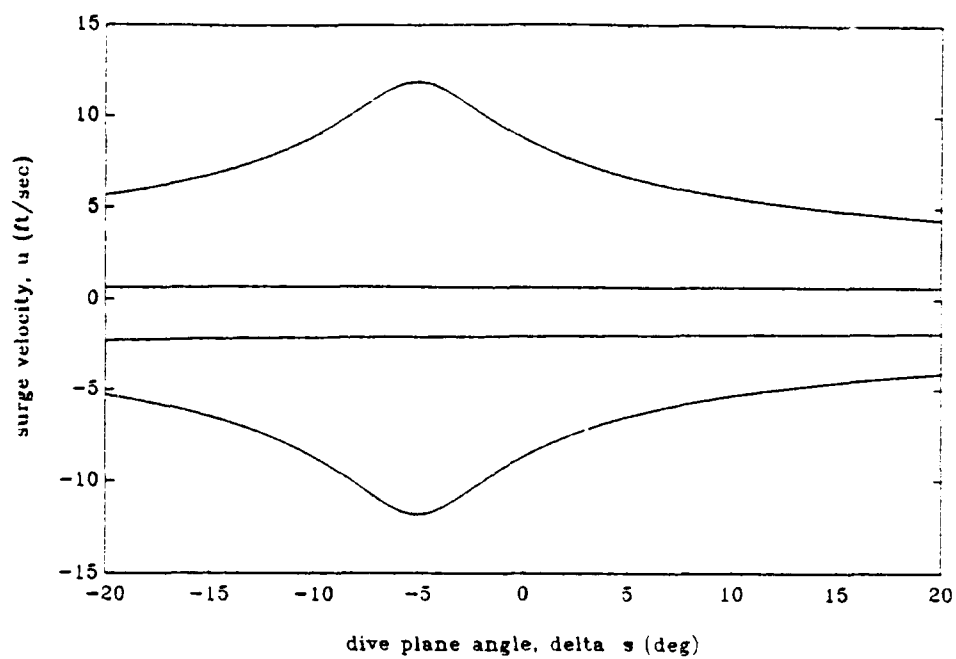
This leads to two possible solutions for u (i.e. $u = \pm \sqrt{u^2}$). The value of w was computed using $w = u(w')$. Combining the two possible solutions of u with the two possible values for w' derived from the quadratic expression lead to four possible combinations of solutions for u and w . The computer program which calculates these four possible solutions is contained in Appendix C. It is an interactive program designed to allow the operator to select the amount of excess buoyancy as a percentage of vehicle weight (δB), the deflection of dive planes in degrees (δ_s), the ratio of bow planes to dive

weight (δB), the deflection of dive planes in degrees (δ_s), the ratio of bow planes to dive planes (δ_b/δ_s), the location of x_{GB} and x_B from body fixed axis origin as a percentage of length, and the location of z_{GB} and z_B from body fixed axis origin in feet.

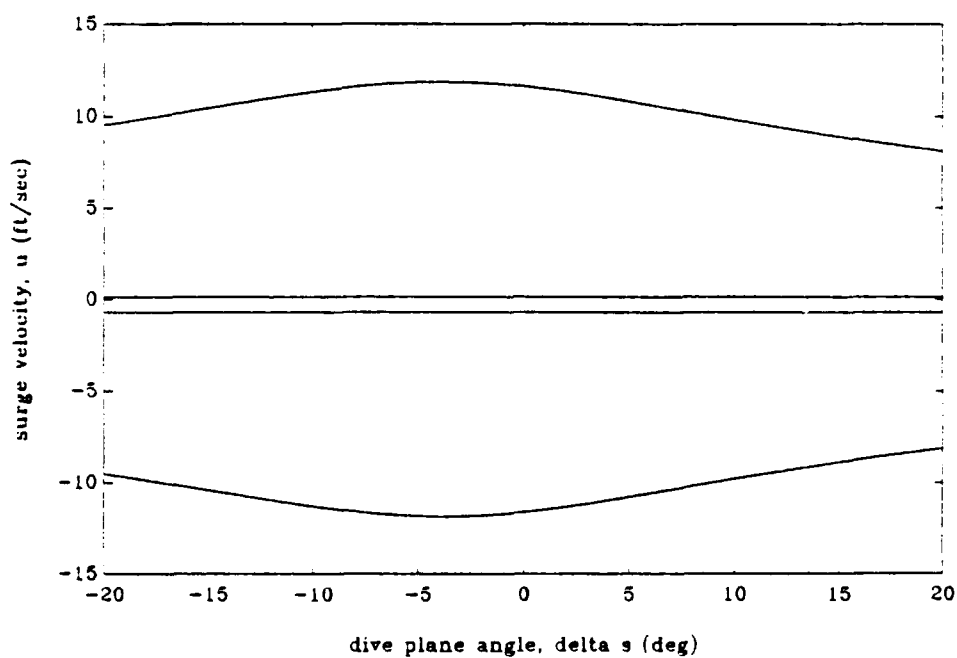
G. STEADY STATE RESULTS

Figures 2, 3, and 4 show typical steady state solutions for surge velocity, heave velocity, and pitch angle as a function of dive plane angle. The two cases shown vary the location of the longitudinal center of buoyancy; for case (a): $x_{GB} = -1\%$ of the vehicle length (L), and for case (b): $x_{GB} = +1\% L$. The following parameters were the same for both cases: excess buoyancy, $\delta B = 2\%$ of the vehicle weight (W); deflection of bow planes, $\delta_b = 0$; location of horizontal and vertical centers of buoyancy, $x_B = z_B = 0$; and location of vertical center of gravity, $z_{GB} = 0.1$ feet.

All runs developed four solutions. For two of the solutions, the magnitude of the surge velocities were large while the magnitudes of the associated heave velocities were relatively small. This has been described by Booth [Ref 2: 297] as "predominantly forward motion". The other two solutions had small surge velocity magnitudes and large heave velocity magnitudes. Booth [Ref 3: 346] referred to this type of motion as "nearly vertical ascents". The positive or negative nature of the velocities are associated with the value of pitch angle. Positive heave velocities are associated with pitch angles greater than 90 degrees. That is the submersible would be ascending in a belly up orientation. Although this steady state analysis computes four possible solutions, it gives no indication as to which of the solutions are stable (if any). Dynamic response and stability criteria will be discussed in the next chapter.

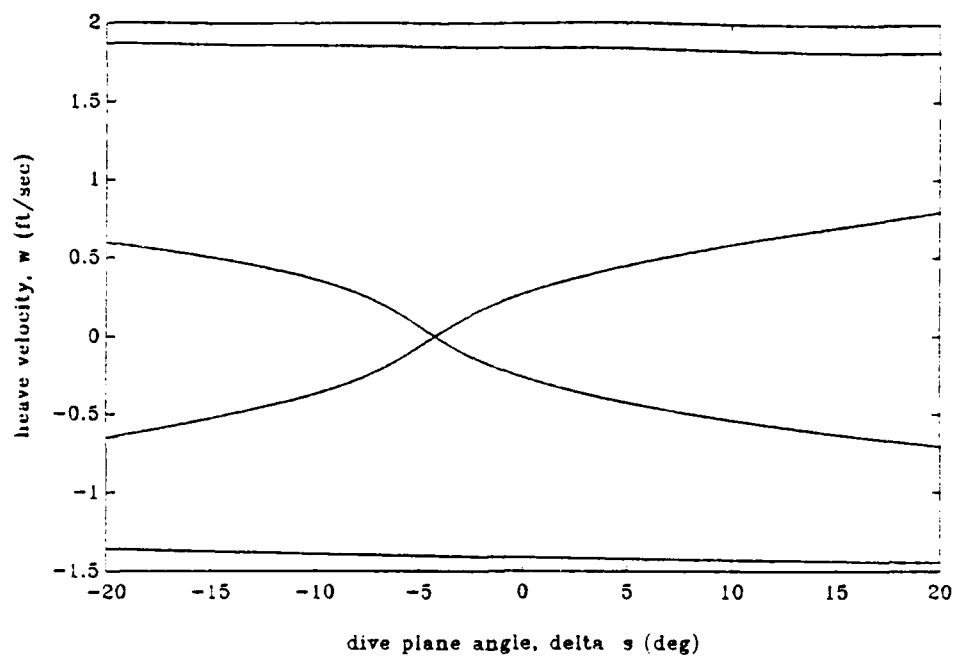


(a) $x_{GB} = -1\%$

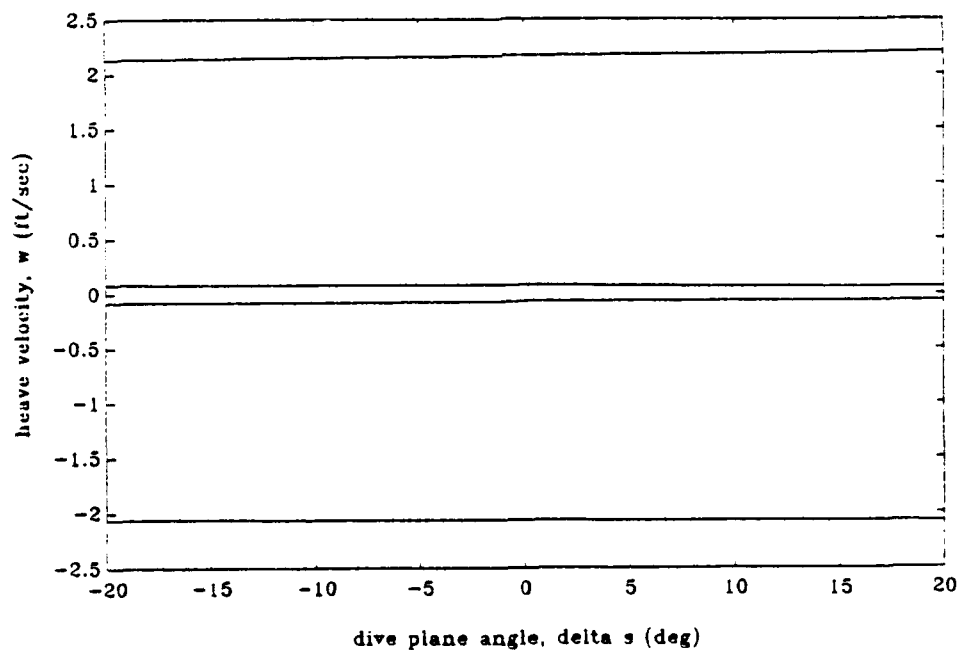


(b) $x_{GB} = 1\%$

Figure 2. Steady State Vertical Plane Solutions for Surge Velocity (u)

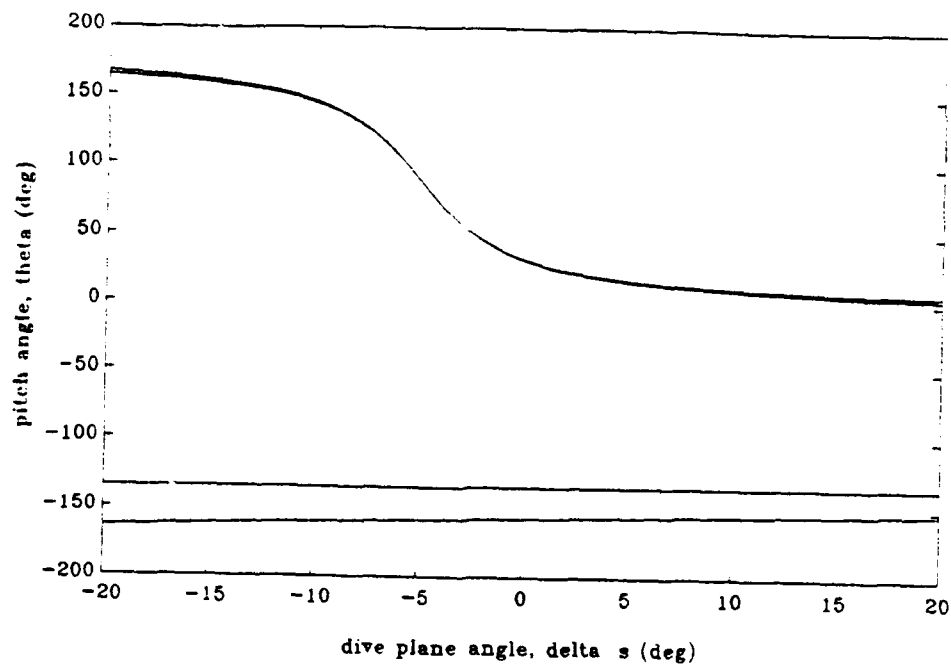


(a) $x_{GB} = -1\%$

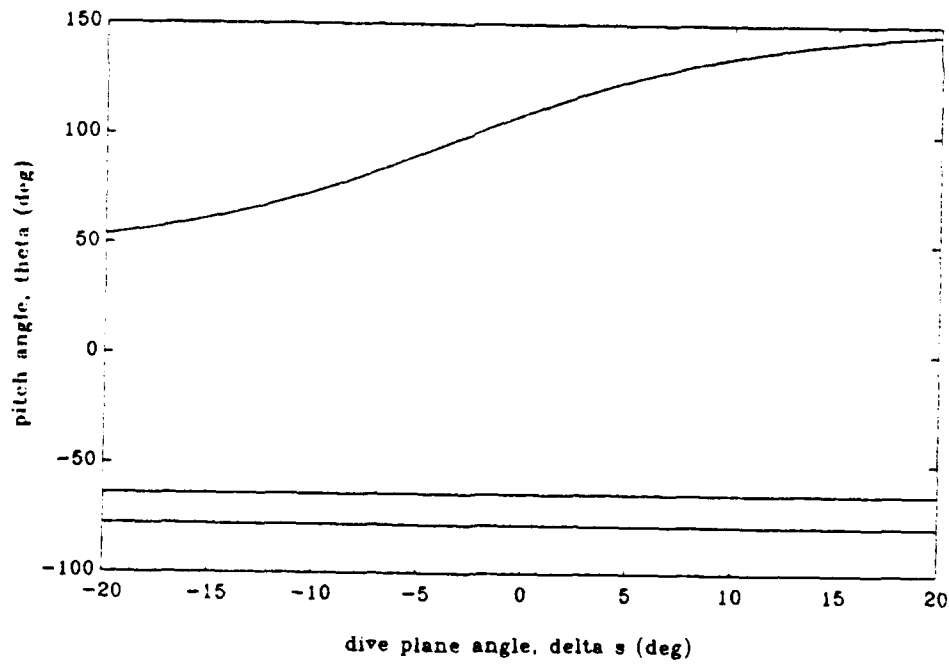


(b) $x_{GB} = 1\%$

Figure 3. Steady State Vertical Plane Solutions for Heave Velocity (w)



(a) $x_{GB} = -1\%$



(b) $x_{GB} = 1\%$

Figure 4. Steady State Vertical Plane Solutions for Pitch Angle (θ)

III. DYNAMIC STABILITY

A. GENERAL

The first portion of this chapter uses the six degree of freedom equations of motion along with the Euler angle rate equations for the derivatives of the angles of pitch and roll (θ and ϕ) to predict the dynamic stability when movement is restricted to the vertical plane. These equations of motion are then linearized around the vertical plane steady state nominal points computed in chapter II. Eigen value analysis is then used to compute the stability of each solution. The second part of the chapter expands this analysis to include all six degrees of freedom and uses the same steady state nominal points to predict the dynamic stability when motion is not restricted to the vertical plane.

B. RESTRICTING MOTION TO THE VERTICAL PLANE

Since motion is restricted to the vertical plane, the body-fixed transverse velocity (v) and its derivative (\dot{v}) are zero. The angles of roll (ϕ) and yaw (ψ), and their derivatives ($\dot{\phi}$ and $\dot{\psi}$) are also zero. We will continue to assume that the lateral center of gravity and the lateral center of buoyancy are on the same centerline plane ($y_G = y_B = 0$), and the rudder is centerlined ($\delta r = 0$).

C. LINEARIZED VERTICAL PLANE EQUATIONS OF MOTION

Substituting these values into the original equations of motion (Appendix A), yields trivial results for three of the six equations: Lateral (sway), Roll, and Yaw. The remaining equations reduce to the following form:

- Longitudinal (Surge) Equation of Motion:

$$\begin{aligned} [m - X_{\dot{u}}] \dot{u} + [m z_G] \dot{q} = & [-C_{D0} + X_{\delta s \delta s} \delta s^2 + X_{\delta b \delta b} \delta b^2] u^2 \\ & + [X_{w \delta s} \delta s + X_{w \delta b} \delta b] uw + [X_{q \delta s} \delta s + X_{q \delta b} \delta b] uq + [X_{ww}] w^2 \\ & + [X_{wq} - m] wq + [X_{qq} + mx_G] q^2 - (W - B) \sin \theta \end{aligned}$$

- Normal (Heave) Equation of Motion:

$$\begin{aligned} [m - Z_{\dot{w}}] \dot{w} - [m x_G + Z_{\dot{q}}] \dot{q} = & [Z_{\delta s} \delta s + Z_{\delta b} \delta b] u^2 \\ & + [Z_w] uw + [m] uq + [mz_G] q^2 \\ & - \int_{x_{tail}}^{x_{nose}} C_{Dz} b(x)(w-xq)^2 \frac{(w-xq)}{U_{cf}(x)} dx + (W - B) \cos \theta \end{aligned}$$

- Pitch Equation of Motion:

$$\begin{aligned} [mz_G] \dot{u} - [M_{\dot{w}} + m x_G] \dot{w} + [I_y - M_{\dot{q}}] \dot{q} = & [M_{\delta s} \delta s + M_{\delta b} \delta b] u^2 + [M_w] uw \\ & + [M_q - mx_G] uq - [mz_G] wq - \int_{x_{tail}}^{x_{nose}} C_{Dz} b(x)(w-xq)^2 \frac{(w-xq)}{U_{cf}(x)} x dx \\ & - (x_G W - x_B B) \cos \theta - (z_G W - z_B B) \sin \theta \end{aligned}$$

These three equations which describe motion restricted to the vertical plane are functions of four variables (u,w,q,θ) plus their derivatives (u̇,ẇ,q̇,θ̇). The equations of motion and the Euler angle rate equation for θ̇ were linearized using the following generalized procedure:

$$B11(i,1)\dot{u} + B11(i,2)\dot{w} + B11(i,3)\dot{q} + B11(i,4)\dot{\theta} = \left[\frac{\partial f_i}{\partial u} \right]_{u_0} u + \left[\frac{\partial f_i}{\partial w} \right]_{w_0} w + \left[\frac{\partial f_i}{\partial q} \right]_{q_0} q + \left[\frac{\partial f_i}{\partial \theta} \right]_{\theta_0} \theta$$

where the B11(i)'s are the constants associated with the derivatives of the variables, the functions f_i represents the right hand side of the nonlinear equations, and i = 1 to 4

identifies the three equations of motion (surge, heave, and yaw) plus the Euler angle rate equation for θ . The partial derivatives were computed as follows:

- **Partial Derivatives of the Longitudinal (Surge) EOM:**

$$\frac{\partial f_1}{\partial u} = 2(-C_{D0} + X_{\delta s \delta s} \delta s^2 + X_{\delta b \delta b} \delta b^2)u_0 + (X_{w \delta s} \delta s + X_{w \delta b} \delta b)w_0 = A11(1,1)$$

$$\frac{\partial f_1}{\partial w} = 2X_{ww}w_0 + (X_{w \delta s} \delta s + X_{w \delta b} \delta b)u_0 = A11(1,2)$$

$$\frac{\partial f_1}{\partial q} = (X_{wq} - m)w_0 + (X_{q \delta s} \delta s + X_{q \delta b} \delta b)u_0 = A11(1,3)$$

$$\frac{\partial f_1}{\partial \theta} = (W - B) \cos \theta_0 = A11(1,4)$$

- **Partial Derivatives of the Normal (Heave) EOM:**

$$\frac{\partial f_2}{\partial u} = Z_w w_0 + 2(Z_{\delta s} \delta s + Z_{\delta b} \delta b)u_0 = A11(2,1)$$

$$\frac{\partial f_2}{\partial w} = u_0 Z_w - 2C_{Dz} A_w |w_0| = A11(2,2)$$

$$\frac{\partial f_2}{\partial q} = u_0(Z_q + m) + 2C_{Dz} A_w x_A |w_0| = A11(2,3)$$

$$\frac{\partial f_2}{\partial \theta} = -(W - B) \sin \theta_0 = A11(2,4)$$

Note on differentiation procedure: The cross-flow velocity term (U_{cf}) was

reduced to $U_{cf}(x) = [(v + xr)^2 + (w - xq)^2]^{0.5} = [w^2]^{0.5} = |w - q|$. Allowing

the integral term in the heave equation to be defined as I_1 , where:

$$I_1 = C_{Dz} \int_{x_{tail}}^{x_{nose}} b(x)(w-xq)|w-xq| dx$$

$$\frac{\partial I_1}{\partial (w-q)} = 2C_{Dz} \int_{x_{tail}}^{x_{nose}} b(x)(w-xq) \text{sign}(w-xq) dx = 2C_{Dz} \int_{x_{tail}}^{x_{nose}} b(x) |w-xq| dx$$

$$\frac{\partial I_1}{\partial w} = \left(\frac{\partial I_1}{\partial (w-q)} \right) \left(\frac{\partial (w-q)}{\partial w} \right) = 2C_{Dz} |w_0| \int_{x_{tail}}^{x_{nose}} b(x) dx = 2C_{Dz} |w_0| A_w$$

$$\frac{\partial I_1}{\partial q} = \left(\frac{\partial I_1}{\partial (w-q)} \right) \left(\frac{\partial (w-q)}{\partial q} \right) = -2C_{Dz} |w_0| \int_{x_{tail}}^{x_{nose}} x b(x) dx = -2C_{Dz} |w_0| x_A A_w$$

• **Partial Derivatives of the Pitch EOM:**

$$\frac{\partial f_3}{\partial u} = 2 (M_{\delta s} \delta s + M_{\delta b} \delta b) u_0 + M_w w_0 = A11(3,1)$$

$$\frac{\partial f_3}{\partial w} = M_w u_0 + 2u_0 C_{Dz} A_w X_A |w_0| = A11(3,2)$$

$$\frac{\partial f_3}{\partial q} = (M_q - m x_G) u_0 - (m z_G) w_0 - 2C_{Dz} I_A |w_0| = A11(3,3)$$

$$\frac{\partial f_3}{\partial \theta} = (x_G W - x_B B) \sin \theta_0 - (z_G W - z_B B) \cos \theta_0 = A11(3,4)$$

Note on differentiation procedure: Again the cross-flow velocity term (U_{cf}) was reduced to $|w-q|$. Allowing the integral term in the pitch equation to be defined

as I_2 , where:

$$I_2 = C_{Dz} \int_{x_{tail}}^{x_{nose}} b(x)(w-xq)|w-xq| x dx$$

$$\frac{\partial I_2}{\partial(w-q)} = 2C_{Dz} \int_{x_{tail}}^{x_{nose}} b(x)|w-xq| x dx$$

$$\frac{\partial I_2}{\partial w} = \left(\frac{\partial I_2}{\partial(w-q)} \right) \left(\frac{\partial(w-q)}{\partial w} \right) = 2C_{Dz}|w_0| \int_{x_{tail}}^{x_{nose}} b(x) x dx = 2C_{Dz}|w_0| x_A A_w$$

$$\frac{\partial I_1}{\partial q} = \left(\frac{\partial I_1}{\partial(w-q)} \right) \left(\frac{\partial(w-q)}{\partial q} \right) = -2C_{Dz}|w_0| \int_{x_{tail}}^{x_{nose}} x^2 b(x) dx = -2C_{Dz}|w_0| I_A$$

- **Partial Derivatives of the Euler Angle Rate Equation for θ :**

$$\frac{\partial f_4}{\partial u} = 0 = A11(4,1)$$

$$\frac{\partial f_4}{\partial w} = 0 = A11(4,2)$$

$$\frac{\partial f_4}{\partial q} = 1 = A11(4,3)$$

$$\frac{\partial f_4}{\partial \theta} = 0 = A11(4,4)$$

The constants corresponding to the derivatives of the four variables (i.e. the left hand side of the four equations) are as follows:

- **Constants Associated with Derivatives for the Longitudinal (Surge) EOM:**

$$\dot{u} \Rightarrow m - X_{\dot{u}} = B11(1,1)$$

$$\dot{q} \Rightarrow m z_G = B11(1,3)$$

$$B11(1,2) = B11(1,4) = 0$$

- **Constants Associated with Derivatives for the Normal (Heave) EOM:**

$$\dot{w} \Rightarrow m - Z_{\dot{w}} = B11(2,2)$$

$$\dot{q} \Rightarrow -(m x_G + Z_{\dot{q}}) = B11(2,3)$$

$$B11(2,1) = B11(2,4) = 0$$

- **Constants Associated with Derivatives for the Pitch EOM:**

$$\dot{u} \Rightarrow m z_G = B11(3,1)$$

$$\dot{w} \Rightarrow -(M_{\dot{w}} + m x_G) = B11(3,2)$$

$$\dot{q} \Rightarrow I_y \cdot \ddot{q} = B_{11}(3,3)$$

$$B_{11}(3,4) = 0$$

- **Constants Associated with Derivatives for the Euler Angle Rate Equation for θ :**

$$\dot{\theta} \Rightarrow 1 = B_{11}(4,4)$$

$$B_{11}(4,1) = B_{11}(4,2) = B_{11}(4,3) = 0$$

These expressions can be arranged in a matrix format to form the linearized equations of motion in the vertical plane about the nominal steady state points. The matrix format is as follows:

$$\mathbf{B}_{11} \times \dot{\mathbf{X}}_1 = \mathbf{A}_{11} \times \mathbf{X}_1$$

where:

$$\mathbf{A}_{11} = \begin{bmatrix} A_{11}(1,1) & A_{11}(1,2) & A_{11}(1,3) & A_{11}(1,4) \\ A_{11}(2,1) & A_{11}(2,2) & A_{11}(2,3) & A_{11}(2,4) \\ A_{11}(3,1) & A_{11}(3,2) & A_{11}(3,3) & A_{11}(3,4) \\ A_{11}(4,1) & A_{11}(4,2) & A_{11}(4,3) & A_{11}(4,4) \end{bmatrix}$$

$$\mathbf{B}_{11} = \begin{bmatrix} B_{11}(1,1) & B_{11}(1,2) & B_{11}(1,3) & B_{11}(1,4) \\ B_{11}(2,1) & B_{11}(2,2) & B_{11}(2,3) & B_{11}(2,4) \\ B_{11}(3,1) & B_{11}(3,2) & B_{11}(3,3) & B_{11}(3,4) \\ B_{11}(4,1) & B_{11}(4,2) & B_{11}(4,3) & B_{11}(4,4) \end{bmatrix}$$

and

$$X1 = \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}$$

D. VERTICAL PLANE COMPUTER PROGRAM DEVELOPMENT

The matrix format of the linearized equations of motion was used in the computer program shown in Appendix D to predict the dynamic stability of the vehicle with motion restricted to the vertical plane. The program is interactive in that it allows the operator to select which of the four data files from the steady state analysis (Chapter II) will be used to define the nominal points for the linearization process. An eigen system subroutine was used to find eigen values and eigen vectors. The program's output was called the degree of stability and only shows the largest real part of all eigen values. The stability criteria is such that the degree of stability must be negative in order for the solution to be stable.

E. VERTICAL PLANE DYNAMIC RESULTS

Of the four possible steady state solutions computed in Chapter II, only one solution from each case yielded stable characteristics. There were some cases in which none of the solutions were stable for certain ranges of parameters. The general trend of the linearized dynamic results are fairly well demonstrated by the two cases discussed in Chapter II. Recalling the parameters of these cases: $\delta B = 2\%$, ratio of bow to stern planes $(\delta_b/\delta_s) = 0$, $z_{GB} = 0.1$ feet, and $x_B = z_B = 0$. The first case placed the longitudinal center of gravity aft of the longitudinal center of buoyancy ($x_{GB} = -1\%$), and the second case placed the longitudinal center of gravity forward ($x_{GB} = +1\%$). Once again dive plane deflection angle (δ_s) was varied from -20 to $+20$ degrees for both cases. Figure 5 shows longitudinal velocity (u) as a function of dive plane angle (δ_s). Case One ($x_{GB} = -1\%$)

showed predominantly forward motion, while case two ($x_{GB} = -1\%$) yielded nearly vertical ascents. Figure 6 shows vertical motion (w) as a function of dive plane angle (δ_s). The results concur with Figure 5, case one shows very little vertical motion while case two demonstrates a larger value. It is interesting to note that vertical motion (w) for case one is positive for dive plane angles between -20 and -4 degrees. The case one values of θ shown in figure 7 for dive plane angles between -20 and -4 degrees concur with this observation. The values of θ greater than 90 degrees indicate the submersible is ascending in the belly up position. The stability in the vertical plane is shown in figure 8. Degree of stability (e) is shown as a function of dive plane angle (δ_s). This figure shows both cases to be stable in the vertical plane.

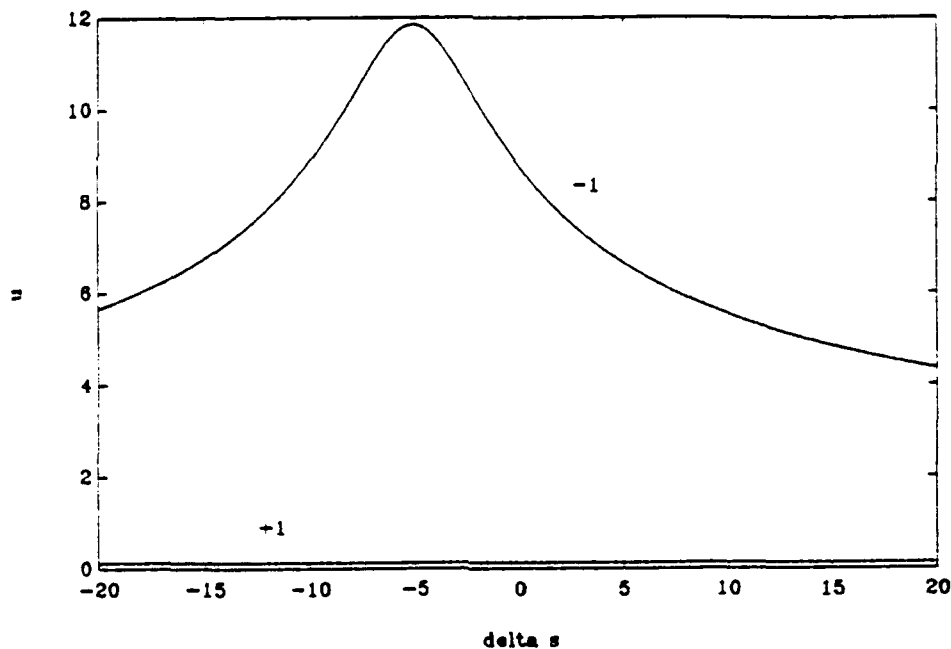


Figure 5. Stable Vertical Plane Solutions for Surge Velocity

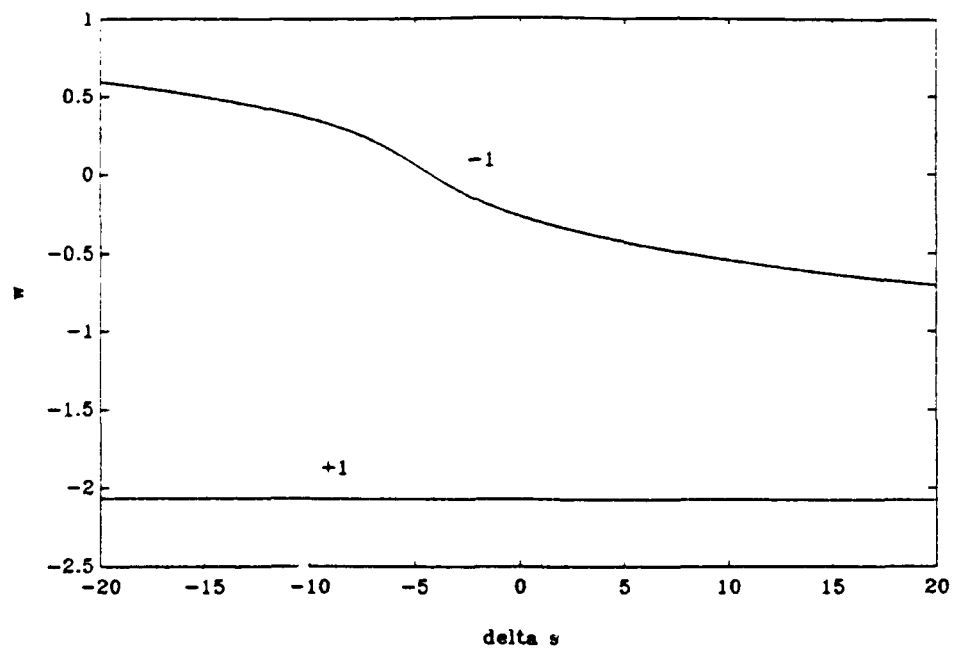


Figure 6. Stable Vertical Plane Solutions for Heave Velocity

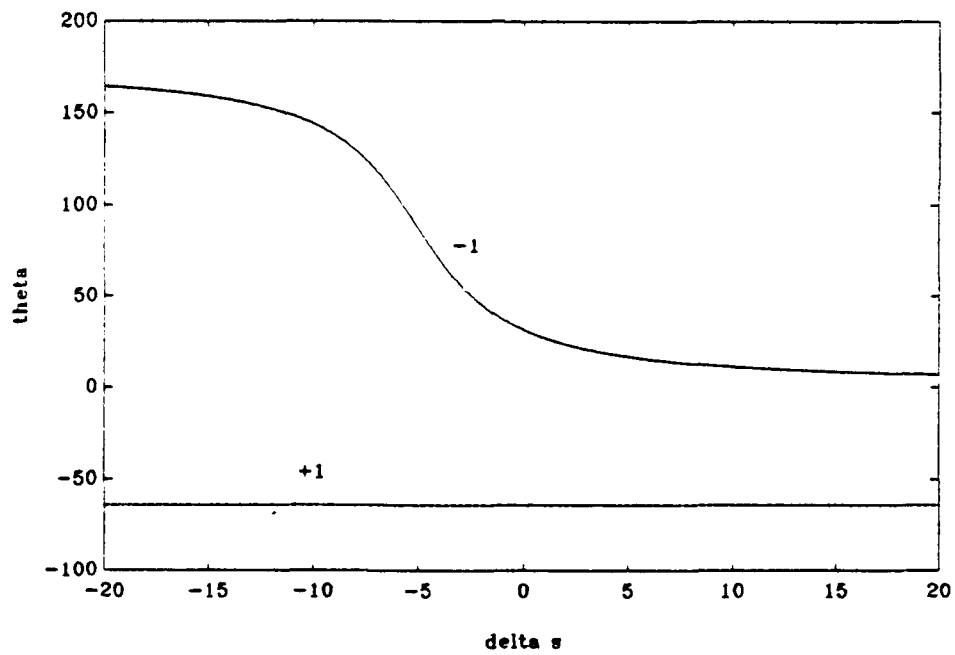


Figure 7. Stable Vertical Plane Solutions for Pitch Angle (θ)

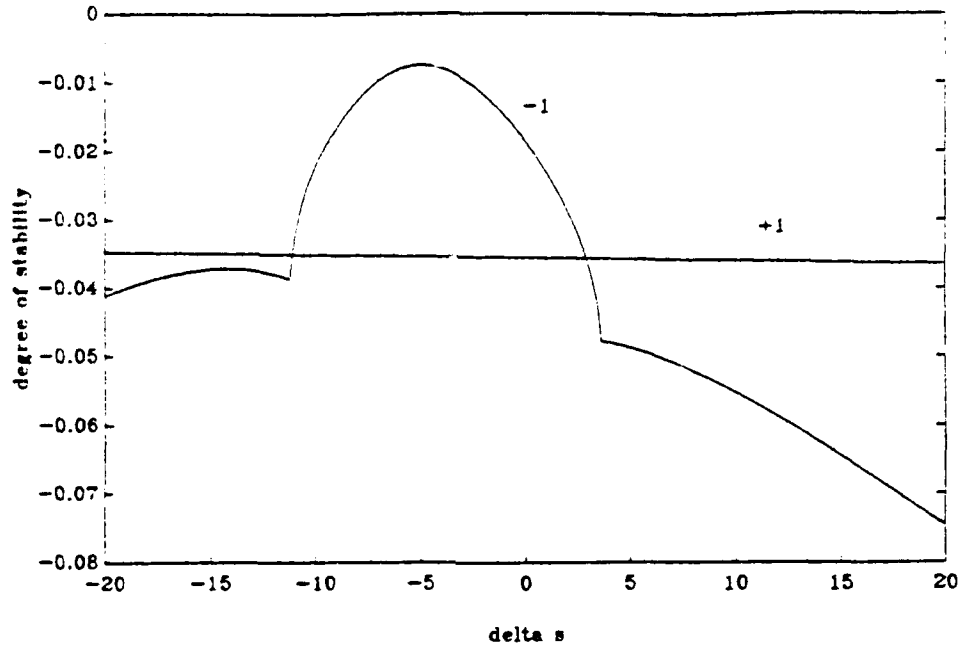


Figure 8. Degree of Stability in the Vertical Plane

F. LINEARIZATION OF FULL EQUATIONS OF MOTION

Referring back to the complete equations of motions shown in Appendix A, these six equations which describe motion for the six degree of freedom system are functions of eight variables $(u, v, w, p, q, r, \phi, \theta)$ plus their derivatives $(\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}, \dot{\phi}, \dot{\theta})$. The six equations of motion (Appendix A) along with the Euler angle rates (Appendix B) were linearized as follows:

$$b_{j1}\dot{u} + b_{j2}\dot{v} + b_{j3}\dot{w} + b_{j4}\dot{p} + b_{j5}\dot{q} + b_{j6}\dot{r} + b_{j7}\dot{\phi} + b_{j8}\dot{\theta} =$$

$$\left[\frac{\partial g_j}{\partial u} \right]_{u_0} u + \left[\frac{\partial g_j}{\partial v} \right]_{v_0} v + \left[\frac{\partial g_j}{\partial w} \right]_{w_0} w + \left[\frac{\partial g_j}{\partial p} \right]_{p_0} p + \left[\frac{\partial g_j}{\partial q} \right]_{q_0} q + \left[\frac{\partial g_j}{\partial r} \right]_{r_0} r + \left[\frac{\partial g_j}{\partial \phi} \right]_{\phi_0} \phi + \left[\frac{\partial g_j}{\partial \theta} \right]_{\theta_0} \theta$$

where the b_j 's are the constants associated with the derivatives of the variables, the functions g_j represents the right hand side of the nonlinear equations, and $j = 1$ to 8

identifies the six equations of motion plus the Euler angle rate equations for ϕ and θ , respectively. The partial derivatives of these equations were computed as follows:

- **Partial Derivatives of Longitudinal (Surge) EOM:**

$$\frac{\partial g_1}{\partial u} = (X_{w\delta s} \delta s + X_{w\delta b} \delta b)w_0 + 2(X_{\delta s\delta s} \delta s^2 + X_{\delta b\delta b} \delta b^2)u_0 - 2C_{D0}u_0 = a11$$

$$\frac{\partial g_1}{\partial w} = 2X_{ww}w_0 + (X_{w\delta s} \delta s + X_{w\delta b} \delta b)u_0 = a13$$

$$\frac{\partial g_1}{\partial q} = -mw_0 + X_{wq}w_0 + (X_{q\delta s} \delta s + X_{q\delta b} \delta b)u_0 = a15$$

$$\frac{\partial g_1}{\partial \theta} = -(W - B) \cos \theta = a18$$

$$\frac{\partial g_1}{\partial v} = 0 = a12$$

$$\frac{\partial g_1}{\partial p} = 0 = a14$$

$$\frac{\partial g_1}{\partial r} = 0 = a16$$

$$\frac{\partial g_1}{\partial \phi} = 0 = a17$$

- **Partial Derivatives of Lateral (Sway) EOM:**

$$\frac{\partial g_2}{\partial v} = Y_v u_0 + Y_{vw} w_0 - C_{Dz} A_w |w_0| = a22$$

$$\frac{\partial g_2}{\partial p} = mw_0 + Y_p u_0 + Y_{wp} w_0 = a24$$

$$\frac{\partial g_2}{\partial r} = -mu_0 + Y_r u_0 + Y_{wr} w_0 - C_{Dz} A_w x_A |w_0| = a26$$

$$\frac{\partial g_2}{\partial \phi} = (W - B) \cos \theta = a27$$

$$\frac{\partial g_2}{\partial u} = 0 = a21 \quad \frac{\partial g_2}{\partial w} = 0 = a23 \quad \frac{\partial g_2}{\partial q} = 0 = a25 \quad \frac{\partial g_2}{\partial \theta} = 0 = a28$$

Note on differentiation procedure: The cross-flow velocity term (U_{cf}) is given

by: $U_{cf}(x) = [(v + xr)^2 + (w - xq)^2]^{0.5}$. The integral term in the sway equation was defined as I_3 , where:

$$\begin{aligned} I_3 &= \int_{x_{tail}}^{x_{nose}} [C_{Dy} h(x) (v + xr)^2 + C_{Dz} b(x) (w - xq)^2] \frac{v + xr}{U_{cf}(x)} dx \\ &= \int_{x_{tail}}^{x_{nose}} (I) \left(\frac{v + xr}{U_{cf}(x)} \right) dx \end{aligned}$$

$$\begin{aligned} \frac{\partial I_3}{\partial v} &= \left(\frac{\partial I}{\partial v} \right) \left(\frac{v + xr}{U_{cf}} \right) + I \frac{\partial}{\partial v} \left(\frac{v + xr}{U_{cf}} \right) = C_{Dz} A_w w_0^2 \left[\frac{U_{cf} - (v + xr) \frac{\partial}{\partial v} U_{cf}}{U_{cf}^2} \right] \\ &= C_{Dz} w_0^2 \frac{|w_0|}{w_0^2} A_w = C_{Dz} |w_0| A_w \end{aligned}$$

$$\begin{aligned} \frac{\partial I_3}{\partial r} &= \left(\frac{\partial I}{\partial r} \right) \left(\frac{v + xr}{U_{cf}} \right) + I \frac{\partial}{\partial r} \left(\frac{v + xr}{U_{cf}} \right) = C_{Dz} A_w w_0^2 \frac{1}{U_{cf}^2} \left[x U_{cf} - (v + xr) \frac{\partial U_{cf}}{\partial r} \right] \\ &= C_{Dz} A_w w_0^2 \frac{x_A |w_0|}{w_0^2} = C_{Dz} A_w x_A |w_0| \end{aligned}$$

$$\frac{\partial I_3}{\partial w} = \left(\frac{\partial I}{\partial w} \right) \left(\frac{v + xr}{U_{cf}} \right) + I \frac{\partial}{\partial w} \left(\frac{v + xr}{U_{cf}} \right) = 0$$

$$\frac{\partial I_3}{\partial q} = \left(\frac{\partial I}{\partial q} \right) \left(\frac{v + xr}{U_{cf}} \right) + I \frac{\partial}{\partial q} \left(\frac{v + xr}{U_{cf}} \right) = 0$$

• **Partial Derivatives of Normal (Heave) EOM:**

$$\frac{\partial g_3}{\partial u} = Z_w w_0 + 2 (Z_{\delta s} \delta s + Z_{\delta b} \delta b) u_0 = a31$$

$$\frac{\partial g_3}{\partial w} = Z_w u_0 - 2 C_{Dz} A_w |w_0| = a33$$

$$\frac{\partial g_3}{\partial q} = m u_0 + Z_q u_0 + 2 C_{Dz} A_w x_A |w_0| = a35$$

$$\frac{\partial g_3}{\partial \theta} = -(W - B) \sin \theta_0 = a38$$

$$\frac{\partial g_3}{\partial v} = 0 = a32$$

$$\frac{\partial g_3}{\partial p} = 0 = a34$$

$$\frac{\partial g_3}{\partial r} = 0 = a36$$

$$\frac{\partial g_3}{\partial \phi} = 0 = a37$$

Note on differentiation procedure: The integral term in the heave equation was defined as I_4 , where:

$$\begin{aligned} I_4 &= \int_{x_{\text{tail}}}^{x_{\text{nose}}} C_{Dy} h(x) (v + x r)^2 + C_{Dz} b(x) (w - x q)^2 \frac{w - x q}{U_{cf}(x)} dx \\ &= \int_{x_{\text{tail}}}^{x_{\text{nose}}} (I) \left(\frac{w - x q}{U_{cf}(x)} \right) dx \end{aligned}$$

$$\begin{aligned} \frac{\partial I_4}{\partial v} &= \left(\frac{\partial I}{\partial v} \right) \left(\frac{w - x q}{U_{cf}} \right) + I \frac{\partial}{\partial v} \left(\frac{w - x q}{U_{cf}} \right) = I \frac{1}{(U_{cf})^2} - (w - x q) \frac{\partial U_{cf}}{\partial v} \\ &= - C_{Dz} A_w w_0^2 \frac{1}{w_0^2} w_0 \frac{\partial U_{cf}}{\partial v} = 0 \quad \text{because} \quad \frac{\partial U_{cf}}{\partial v} = 0 \end{aligned}$$

$$\begin{aligned}
\frac{\partial I_4}{\partial w} &= \left(\frac{\partial I}{\partial w} \right) \left(\frac{w - xq}{U_{cf}} \right) + I \frac{\partial}{\partial w} \left(\frac{w - xq}{U_{cf}} \right) \\
&= 2 C_{Dz} A_w (w - xq) \frac{w - xq}{U_{cf}} + I \frac{1}{U_{cf}^2} \left[U_{cf} - (w - xq) \frac{\partial U_{cf}}{\partial w} \right] \\
&= 2 C_{Dz} A_w w_0^2 \frac{1}{|w_0|} + C_{Dz} A_w w_0^2 \frac{1}{w_0^2} \left[|w_0| - w_0 \frac{w_0}{|w_0|} \right] = 2 C_{Dz} A_w |w_0| \\
\text{because } \frac{\partial U_{cf}}{\partial w} &= \frac{1}{2} U_{cf}(x) = \left[(v + xr)^2 + (w - xq)^2 \right]^{0.5} 2(w - xq) = \frac{w}{|w|}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial I_4}{\partial q} &= \left(\frac{\partial I}{\partial q} \right) \left(\frac{w - xq}{U_{cf}} \right) + I \frac{\partial}{\partial q} \left(\frac{w - xq}{U_{cf}} \right) = C_{Dz} A_w w_0^2 \frac{1}{U_{cf}^2} \left[x U_{cf} - (w - xq) \frac{\partial U_{cf}}{\partial r} \right] \\
&= C_{Dz} A_w w_0^2 \frac{x_A |w_0|}{w_0^2} = C_{Dz} A_w x_A |w_0|
\end{aligned}$$

$$\frac{\partial I_4}{\partial r} = 0$$

• **Partial Derivatives of Roll EOM:**

$$\frac{\partial g_4}{\partial v} = K_v u_0 + K_{vw} w_0 = a42$$

$$\frac{\partial g_4}{\partial p} = -m z_G w_0 + K_p u_0 + K_{wp} w_0 = a44$$

$$\frac{\partial g_4}{\partial r} = m z_G u_0 + K_r u_0 + K_{wr} w_0 = a46$$

$$\frac{\partial g_4}{\partial \phi} = - (z_G W - z_B B) \cos \theta_0 = a47$$

$$\frac{\partial g_4}{\partial u} = 0 = a41$$

$$\frac{\partial g_4}{\partial w} = 0 = a43$$

$$\frac{\partial g_4}{\partial q} = 0 = a45$$

$$\frac{\partial g_4}{\partial \theta} = 0 = a48$$

• **Partial Derivatives of Pitch EOM:**

$$\frac{\partial g_5}{\partial u} = M_w w_0 + 2 (M_{\delta s} \delta s + M_{\delta b} \delta b) u_0 = a51$$

$$\frac{\partial g_5}{\partial w} = M_w u_0 + 2 C_{Dz} A_w x_A |w_0| = a53$$

$$\frac{\partial g_5}{\partial q} = -m x_G u_0 - m z_G w_0 + M_q u_0 - 2 C_{Dz} I_A |w_0| = a55$$

$$\frac{\partial g_5}{\partial \theta} = (x_G W - x_B B) \sin \theta_0 - (z_G W - z_B B) \cos \theta_0 = a58$$

$$\frac{\partial g_5}{\partial v} = 0 = a52$$

$$\frac{\partial g_5}{\partial p} = 0 = a54$$

$$\frac{\partial g_5}{\partial r} = 0 = a56$$

$$\frac{\partial g_5}{\partial \phi} = 0 = a57$$

Note on differentiation procedure: The integral term in the pitch equation was defined as I_5 , where:

$$\begin{aligned} I_5 &= \int_{x_{tail}}^{x_{nose}} \left[C_{Dy} h(x) (v + x r)^2 + C_{Dz} h(x) (w - x q)^2 \right] \frac{w - x q}{U_{cl}(x)} x dx \\ &= \int_{x_{tail}}^{x_{nose}} (I) \left(\frac{w - x q}{U_{cl}(x)} \right) x dx = \int_{x_{tail}}^{x_{nose}} I_4 x dx \end{aligned}$$

$$\frac{\partial I_5}{\partial w} = \frac{\partial I_4}{\partial w} x_A = 2 C_{Dz} A_w x_A |w_0|$$

$$\frac{\partial I_5}{\partial q} = \frac{\partial I_4}{\partial q} x_A = C_{Dz} A_w x_A^2 |w_0| = C_{Dz} I_A |w_0|$$

$$\frac{\partial I_5}{\partial v} = \frac{\partial I_4}{\partial v} x_A = 0$$

$$\frac{\partial I_5}{\partial r} = \frac{\partial I_4}{\partial r} x_A = 0$$

- **Partial Derivatives of Yaw EOM:**

$$\frac{\partial g_6}{\partial v} = N_v u_0 + N_{vw} w_0 - C_{Dz} A_w x_A |w_0| = a62$$

$$\frac{\partial g_6}{\partial p} = m x_G w_0 + N_p u_0 + N_{wp} w_0 = a64$$

$$\frac{\partial g_6}{\partial r} = - m x_G u_0 + N_r u_0 + N_{wr} w_0 - C_{Dz} I_A |w_0| = a66$$

$$\frac{\partial g_6}{\partial \phi} = (x_G W - x_B B) \cos \theta_0 = a67$$

$$\frac{\partial g_6}{\partial u} = 0 = a61$$

$$\frac{\partial g_6}{\partial w} = 0 = a63$$

$$\frac{\partial g_6}{\partial q} = 0 = a65$$

$$\frac{\partial g_6}{\partial \theta} = 0 = a68$$

Note on differentiation procedure: The integral term in the yaw equation was defined as I_6 , where:

$$\begin{aligned} I_6 &= \int_{x_{tail}}^{x_{nose}} \left[C_{Dy} h(x) (v+xr)^2 + C_{Dz} b(x) (w-xq)^2 \right] \frac{v+xr}{U_{cf}(x)} x dx \\ &= \int_{x_{tail}}^{x_{nose}} (I) \left(\frac{v+xr}{U_{cf}(x)} \right) x dx = \int_{x_{tail}}^{x_{nose}} I_3 x dx \end{aligned}$$

$$\frac{\partial I_6}{\partial v} = \frac{\partial I_3}{\partial v} x_A = C_{Dz} |w_0| A_w x_A$$

$$\frac{\partial I_3}{\partial r} = \frac{\partial I_3}{\partial r} x_A = C_{Dz} A_w x_A^2 |w_0| = C_{Dz} I_A |w_0|$$

$$\frac{\partial I_6}{\partial w} = \frac{\partial I_3}{\partial w} x_A = 0$$

$$\frac{\partial I_6}{\partial q} = \frac{\partial I_3}{\partial q} x_A = 0$$

- **Partial Derivatives of Euler Angle Rate for ϕ :**

$$\frac{\partial g_7}{\partial p} = 1 = a74$$

$$\frac{\partial g_7}{\partial r} = \tan \theta_0 = a76$$

$$\frac{\partial g_7}{\partial u} = 0 = a71$$

$$\frac{\partial g_7}{\partial v} = 0 = a72$$

$$\frac{\partial g_7}{\partial w} = 0 = a73$$

$$\frac{\partial g_7}{\partial q} = 0 = a75$$

$$\frac{\partial g_7}{\partial \phi} = 0 = a77$$

$$\frac{\partial g_7}{\partial \theta} = 0 = a78$$

- **Partial Derivatives of Euler Angle Rate for θ :**

$$\frac{\partial g_8}{\partial q} = 1 = a86$$

$$\frac{\partial g_8}{\partial u} = 0 = a81$$

$$\frac{\partial g_8}{\partial v} = 0 = a82$$

$$\frac{\partial g_8}{\partial w} = 0 = a83$$

$$\frac{\partial g_8}{\partial q} = 0 = a85$$

$$\frac{\partial g_8}{\partial r} = 0 = a86$$

$$\frac{\partial g_8}{\partial \phi} = 0 = a87$$

$$\frac{\partial g_8}{\partial \theta} = 0 = a88$$

The constants corresponding to the derivatives of the eight variables (i.e. the left hand side of the eight equations) are as follows:

- **Constants Associated with Derivatives for the Longitudinal (Surge) EOM:**

$$\dot{u} \Rightarrow (m - X_{\dot{u}}) = b11 \qquad \dot{q} \Rightarrow (mz_G) = b15$$

$$b12 = b13 = b14 = b16 = b17 = b18 = 0$$

- **Constants Associated with Derivatives for the Lateral (Sway) EOM:**

$$\dot{v} \Rightarrow (m - Y_{\dot{v}}) = b22 \qquad \dot{p} \Rightarrow (-mz_G - Y_{\dot{p}}) = b24$$

$$\dot{r} \Rightarrow (mx_G - Y_{\dot{r}}) = b26$$

$$b21 = b23 = b25 = b27 = b28 = 0$$

- **Constants Associated with Derivatives for the Normal (Heave) EOM:**

$$\dot{w} \Rightarrow (m - Z_{\dot{w}}) = b33 \qquad \dot{q} \Rightarrow (-mx_G - Z_{\dot{q}}) = b35$$

$$b31 = b32 = b34 = b36 = b37 = b38$$

- **Constants Associated with Derivatives for the Roll EOM:**

$$\dot{v} \Rightarrow (-mz_G - K_{\dot{v}}) = b42 \qquad \dot{r} \Rightarrow (-K_{\dot{r}}) = b46$$

$$\dot{p} \Rightarrow (I_x - K_{\dot{p}}) = b44$$

$$b41 = b43 = b45 = b47 = b48$$

- **Constants Associated with Derivatives for the Pitch EOM:**

$$\dot{u} \Rightarrow (mz_G) = b51 \qquad \dot{w} \Rightarrow (-mx_G - M_{\dot{w}}) = b53$$

$$\dot{q} \Rightarrow (I_y - M_{\dot{q}}) = b55$$

$$b52 = b54 = b56 = b57 = b58$$

- **Constants Associated with Derivatives for the Yaw EOM:**

$$\dot{v} \Rightarrow (mx_G - N_{\dot{v}}) = b62 \qquad \dot{p} \Rightarrow (-N_{\dot{p}}) = b64$$

$$\dot{r} \Rightarrow (I_z - N_{\dot{r}}) = b66$$

$$b61 = b63 = b65 = b67 = b68 = 0$$

- **Constants Associated with Derivatives for the Euler Angle Rate**

Equation for $\dot{\phi}$:

$$\dot{\phi} \Rightarrow 1 = b77$$

$$b71 = b72 = b73 = b74 = b75 = b76 = b78 = 0$$

- **Constants Associated with the Derivatives for the Euler Angle**

Rate Equation for $\dot{\theta}$:

$$\dot{\theta} \Rightarrow 1 = b88$$

$$b81 = b82 = b83 = b84 = b85 = b86 = b87 = 0$$

These expressions can be arranged in a matrix format to define the dynamic equations of motion for the six degree of freedom system linearized about the nominal steady state points. The matrix format is $\mathbf{B} \times \dot{\mathbf{X}} = \mathbf{A} \times \mathbf{X}$, where:

$$\mathbf{A} = \begin{matrix} & \begin{matrix} a11 & a12 & a13 & a14 & a15 & a16 & a17 & a18 \end{matrix} \\ \begin{matrix} a21 & a22 & a23 & a24 & a25 & a26 & a27 & a28 \\ a31 & a32 & a33 & a34 & a35 & a36 & a37 & a38 \\ a41 & a42 & a43 & a44 & a45 & a46 & a47 & a48 \\ a51 & a52 & a53 & a54 & a55 & a56 & a57 & a58 \\ a61 & a62 & a63 & a64 & a65 & a66 & a67 & a68 \\ a71 & a72 & a73 & a74 & a75 & a76 & a77 & a78 \\ a81 & a82 & a83 & a84 & a85 & a86 & a87 & a88 \end{matrix} \end{matrix}$$

$$= \begin{matrix} & \begin{matrix} a11 & 0 & a13 & 0 & a15 & 0 & 0 & a18 \end{matrix} \\ \begin{matrix} 0 & a22 & 0 & a24 & 0 & a26 & a27 & 0 \\ a31 & 0 & a33 & 0 & a35 & 0 & 0 & a38 \\ 0 & a42 & 0 & a44 & 0 & a46 & a47 & a0 \\ a51 & 0 & a53 & 0 & a55 & 0 & 0 & a58 \\ 0 & a62 & 0 & a64 & 0 & a66 & a67 & 0 \\ 0 & 0 & 0 & 1 & 0 & a76 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{matrix} \end{matrix}$$

$$\mathbf{B} = \begin{bmatrix}
 b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} & b_{17} & b_{18} \\
 b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} & b_{27} & b_{28} \\
 b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} & b_{37} & b_{38} \\
 b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} & b_{47} & b_{48} \\
 b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & b_{56} & b_{57} & b_{58} \\
 b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} & b_{67} & b_{68} \\
 b_{71} & b_{72} & b_{73} & b_{74} & b_{75} & b_{76} & b_{77} & b_{78} \\
 b_{81} & b_{82} & b_{83} & b_{84} & b_{85} & b_{86} & b_{87} & b_{88}
 \end{bmatrix}$$

$$= \begin{bmatrix}
 b_{11} & 0 & 0 & 0 & b_{15} & 0 & 0 & 0 \\
 0 & b_{22} & 0 & b_{24} & 0 & b_{26} & 0 & 0 \\
 0 & 0 & b_{33} & 0 & b_{35} & 0 & 0 & 0 \\
 0 & b_{42} & 0 & b_{44} & 0 & b_{46} & 0 & 0 \\
 b_{51} & 0 & b_{53} & 0 & b_{55} & 0 & 0 & 0 \\
 0 & b_{62} & 0 & b_{64} & 0 & b_{66} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
 \end{bmatrix}$$

and

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \phi \\ \theta \end{bmatrix}$$

These matrices may be reordered such that they will be of the form:

$$\begin{bmatrix} \mathbf{B11} & 0 \\ 0 & \mathbf{B22} \end{bmatrix} \times \begin{bmatrix} \mathbf{X1} \\ \mathbf{X2} \end{bmatrix} = \begin{bmatrix} \mathbf{A11} & 0 \\ 0 & \mathbf{A22} \end{bmatrix} \times \begin{bmatrix} \mathbf{X1} \\ \mathbf{X2} \end{bmatrix}$$

This is accomplished by rewriting the **X** matrix such that **X1** is the same matrix used in the vertical plane analysis:

$$\mathbf{X} = \begin{bmatrix} x1 \\ x3 \\ x5 \\ x8 \\ x4 \\ x7 \\ x2 \\ x6 \end{bmatrix} = \begin{bmatrix} u \\ w \\ q \\ \theta \\ p \\ \phi \\ v \\ r \end{bmatrix} = \begin{bmatrix} \mathbf{X1} \\ \mathbf{X2} \end{bmatrix}$$

The **A** matrix is restructured into a matrix containing four 4×4 matrices with the **A12** and **A21** matrices containing only the zero element.

$$\mathbf{A} = \begin{bmatrix} a11 & a13 & a15 & a18 & 0 & 0 & 0 & 0 \\ a31 & a33 & a35 & a38 & 0 & 0 & 0 & 0 \\ a51 & a53 & a55 & a58 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a44 & a47 & a42 & a46 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & a76 \\ 0 & 0 & 0 & 0 & a24 & a27 & a22 & a26 \\ 0 & 0 & 0 & 0 & a64 & a67 & a62 & a66 \end{bmatrix} = \begin{bmatrix} \mathbf{A11} & 0 \\ 0 & \mathbf{A22} \end{bmatrix}$$

Similarly, **B** is restructured into a matrix containing four 4×4 matrices with the **B12** and **B21** matrices containing only the zero element.

$$\mathbf{B} = \begin{bmatrix} b_{11} & 0 & b_{15} & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{33} & b_{35} & 0 & 0 & 0 & 0 & 0 \\ b_{51} & b_{53} & b_{55} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{44} & 0 & b_{42} & b_{46} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{24} & 0 & b_{22} & b_{26} \\ 0 & 0 & 0 & 0 & b_{64} & 0 & b_{62} & b_{66} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{11} & 0 \\ 0 & \mathbf{B}_{22} \end{bmatrix}$$

As discussed, the **X1** matrix established to describe the linearized dynamic stability in the vertical plane is the same as the **X1** matrix within **X**. In addition, the **A11** and **B11** matrices from the vertical plane analysis are also identical to those in **X**. That is:

$$\mathbf{A}_{11} = \begin{bmatrix} A_{11}(1,1) & A_{11}(1,2) & A_{11}(1,3) & A_{11}(1,4) \\ A_{11}(2,1) & A_{11}(2,2) & A_{11}(2,3) & A_{11}(2,4) \\ A_{11}(3,1) & A_{11}(3,2) & A_{11}(3,3) & A_{11}(3,4) \\ A_{11}(4,1) & A_{11}(4,2) & A_{11}(4,3) & A_{11}(4,4) \end{bmatrix} = \begin{bmatrix} a_{44} & a_{47} & a_{42} & a_{46} \\ 1 & 0 & 0 & a_{76} \\ a_{24} & a_{27} & a_{22} & a_{26} \\ a_{64} & a_{67} & a_{62} & a_{66} \end{bmatrix}$$

$$\mathbf{B}_{11} = \begin{bmatrix} B_{11}(1,1) & B_{11}(1,2) & B_{11}(1,3) & B_{11}(1,4) \\ B_{11}(2,1) & B_{11}(2,2) & B_{11}(2,3) & B_{11}(2,4) \\ B_{11}(3,1) & B_{11}(3,2) & B_{11}(3,3) & B_{11}(3,4) \\ B_{11}(4,1) & B_{11}(4,2) & B_{11}(4,3) & B_{11}(4,4) \end{bmatrix} = \begin{bmatrix} b_{44} & 0 & b_{42} & b_{46} \\ 0 & 1 & 0 & 0 \\ b_{24} & 0 & b_{22} & b_{26} \\ b_{64} & 0 & b_{62} & b_{66} \end{bmatrix}$$

The **A22**, **B22**, and **X2** matrices represent the additional equations necessary to describe the linearized motion for all six degrees of freedom; henceforth referred to as the horizontal

plane contributions. The eigen function for the six degree of freedom model is computed by taking the determinant as follows:

$$\det \begin{bmatrix} \mathbf{A11} - \lambda \mathbf{B11} & 0 \\ 0 & \mathbf{A22} - \lambda \mathbf{B22} \end{bmatrix} = 0 \Rightarrow (\det [\mathbf{A11} - \lambda \mathbf{B11}]) (\det [\mathbf{A22} - \lambda \mathbf{B22}]) = 0$$

Since the eigen function will be the product of these two determinants, the resulting eigen values will merely be the union of the vertical plane eigen values and the horizontal plane eigen values. The significance of reducing the eigen value calculation from an 8 by 8 matrix problem to two 4 by 4 matrix problems is not in the computation time saved. But rather in fact that now the horizontal and vertical stabilities have been separated and identified.

G. COMPUTER PROGRAM DEVELOPMENT

The matrix format of the linearized dynamic response equations associated with motion in the horizontal plane was added to the computer program developed previously (Appendix D). Once again, the program is interactive in that it allows the operator to select which of the four data files from the steady state analysis (Chapter II) will be used to define the nominal points for the linearization process. An eigen system subroutine was used to find eigen values and eigen vectors. Two outputs were added to the program. First, the degree of stability in the horizontal plane, and next the degree of stability of both planes (i.e., the union of the vertical and horizontal degrees of stability). Reminder: degree of stability must be negative in order for the solution to be stable.

H. DYNAMIC STABILITY SOLUTIONS

Continuing with the same cases from part E, the horizontal stability for the two cases are shown in figure 9. Case two ($x_{GB} = +1\%$) is stable in the horizontal plane for all

values of dive plane angle (δ_s). Whereas, case one ($x_{GB} = -1\%$) is unstable in the horizontal plane for dive plane angle (δ_s) between -20 and -9 degrees. From figure 7, this corresponds to values of θ greater than 140 degrees. This indicates that the vehicle is stable (even in the horizontal plane) for values of θ greater than 90 degrees. A submersible with an angle of pitch greater than 90 degrees will have a negative metacentric height and will therefore be statically unstable. However, the results shown in figures 7 and 9 indicate that the vehicle will remain dynamically stable in such a condition. This 'inverted pendulum' type stability will be further investigated during the numerical integration analysis (Chapter IV) to see if hydrodynamic and drag forces on the vehicle can actually cause this to occur. Figure 10 shows the combined stabilities for the horizontal and vertical planes. It should be noted that in general the horizontal plane dictated stability for the cases considered.

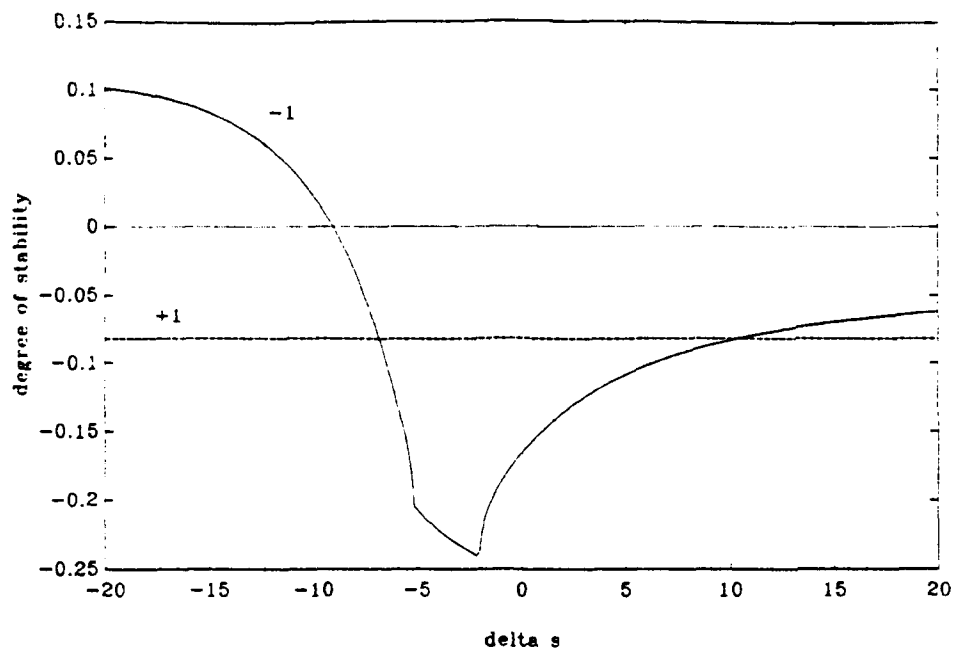


Figure 9. Degree of Stability in the Horizontal Plane

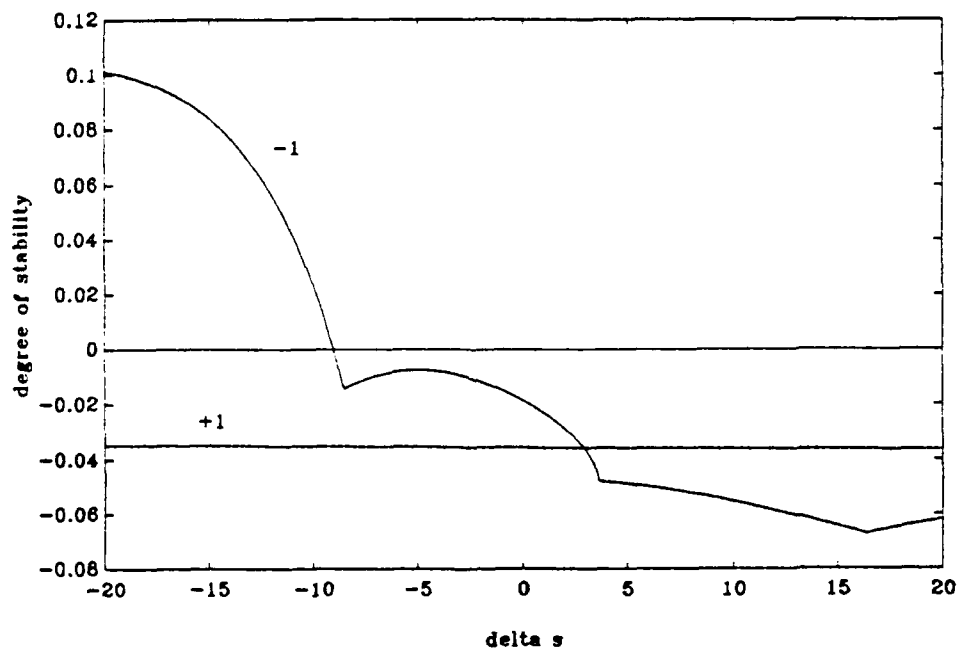


Figure 10. Degree of Stability in Both (Horizontal and Vertical) Planes

Because the real part of the computed eigen values must be negative in order for a solution to be stable, it is easy to identify unstable solutions. However, the measure of stability for a stable solution is harder to quantify. The magnitudes shown thus far for degree of stability seem very small. Does this mean that the solutions are not very stable? In an attempt to answer this question, the values for degree of stability for a neutrally buoyant submersible ($\delta B = 0$ or $W = B$) were computed. Figure 11 shows the degree of stability in (a) the horizontal plane and (b) the vertical plane as a function of surge velocity. This figure shows that the magnitude of the values for degree of stability for this neutrally buoyant case are indeed of the same order of magnitude as the positively buoyant cases discussed earlier.

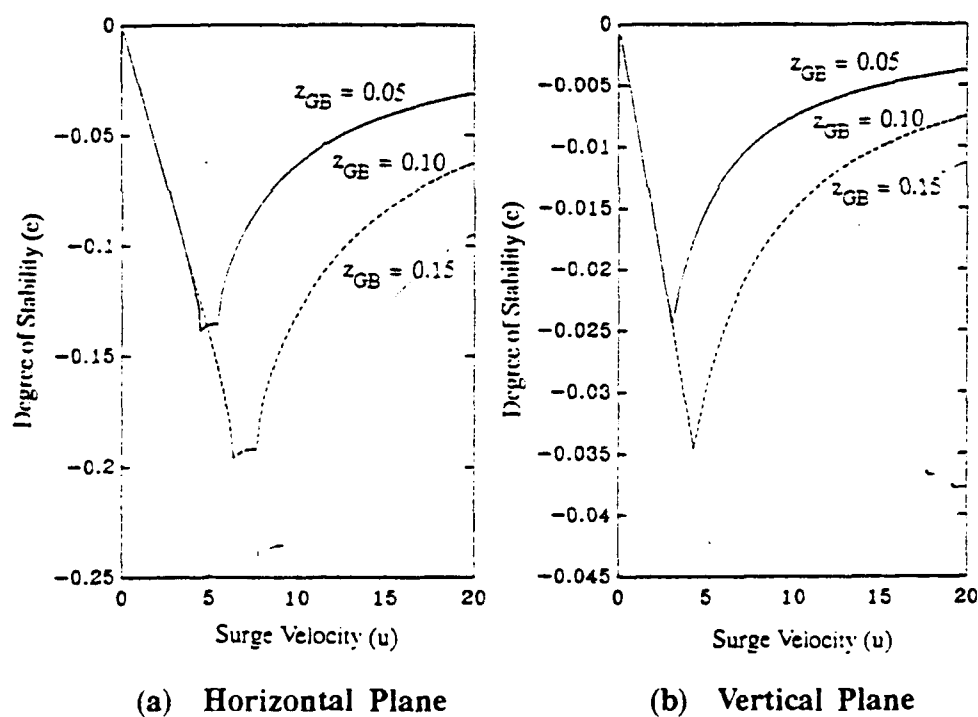


Figure 11. Degree of Stability as a Function of Surge Velocity (u) for a Neutrally Buoyant Submersible

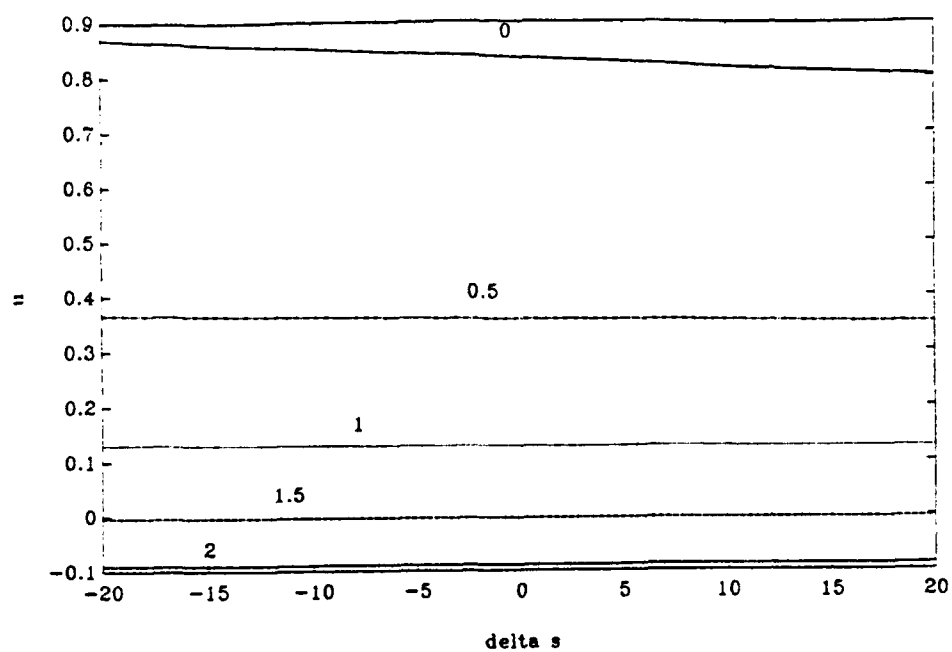
IV. SIMULATIONS AND DISCUSSION OF RESULTS

A. DYNAMIC STABILITY ANALYSIS RESULTS

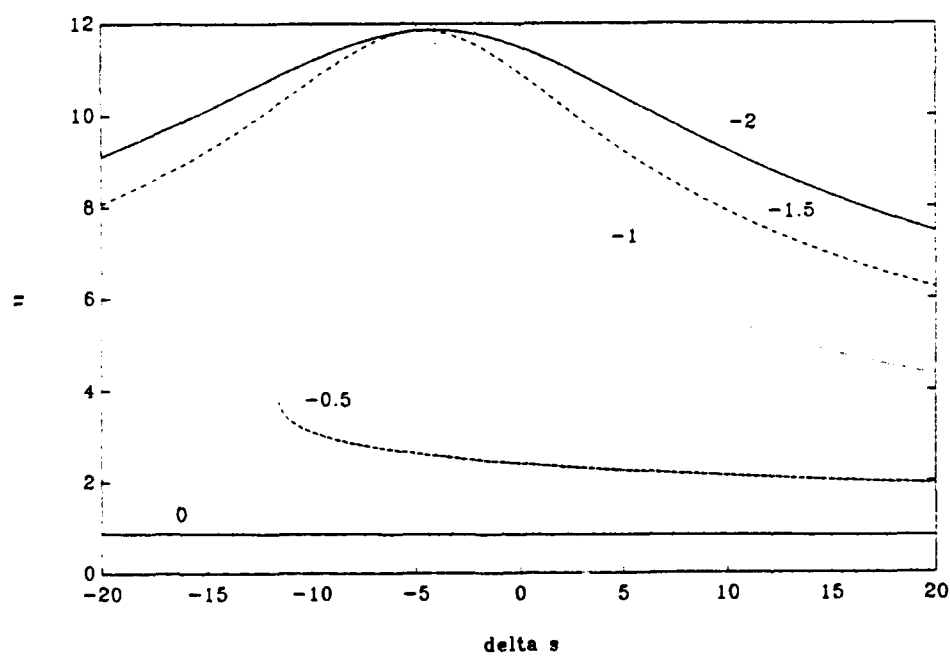
The dynamic stability analysis included in this Chapter considers stability for both planes (i.e. the analysis no longer breaks stability down by horizontal or vertical plane). As mentioned in Chapter III, horizontal plane stability generally dictates the stability of the vehicle.

1. Variations in Longitudinal Center of Gravity (x_{GB})

Figures 12 through 15 show how changing the longitudinal center of gravity (x_{GB}) effects the dynamic response of the submersible. For these cases, the amount of excess buoyancy (δB) is two percent of weight (W), the bow plane deflection angle (δ_b) is zero, the vertical center of gravity (z_{GB}) is 0.1 feet, and the longitudinal and vertical centers of buoyancy (x_B and z_B) are zero. The longitudinal center of gravity (x_{GB}) is varied from - 2 to + 2 percent of vehicle length. When the longitudinal center of gravity (x_{GB}) is greater than zero, there are stable solutions for the full range of dive plane angles. However, this is not true when the longitudinal center of gravity (x_{GB}) is less than zero. The range of stable solutions is restricted when $x_{GB} = - 0.5$ and $- 1.0$.

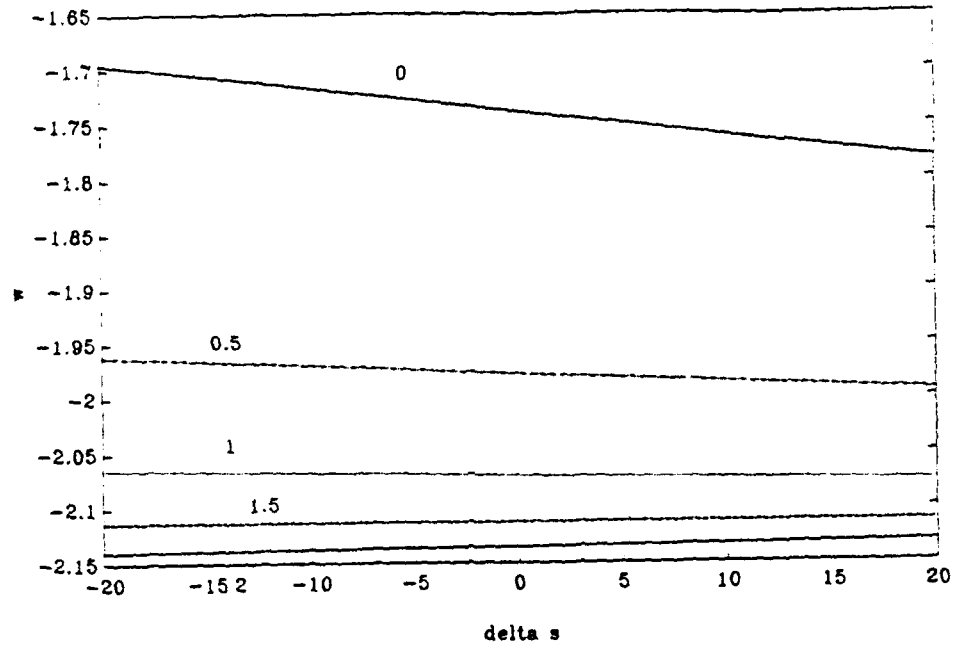


(a) $x_{GB} = 0, 0.5, 1.0, 1.5$ and 2.0%

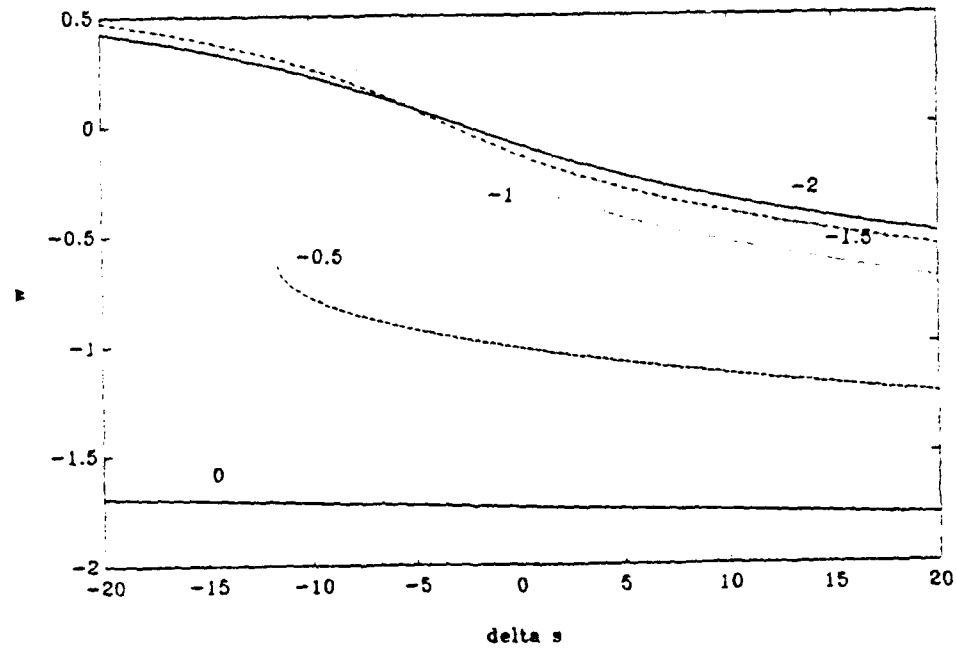


(b) $x_{GB} = 0, -0.5, -1.0, -1.5$ and -2.0%

Figure 12. Stable Surge Velocity (u) Solutions for Variations in x_{GB}

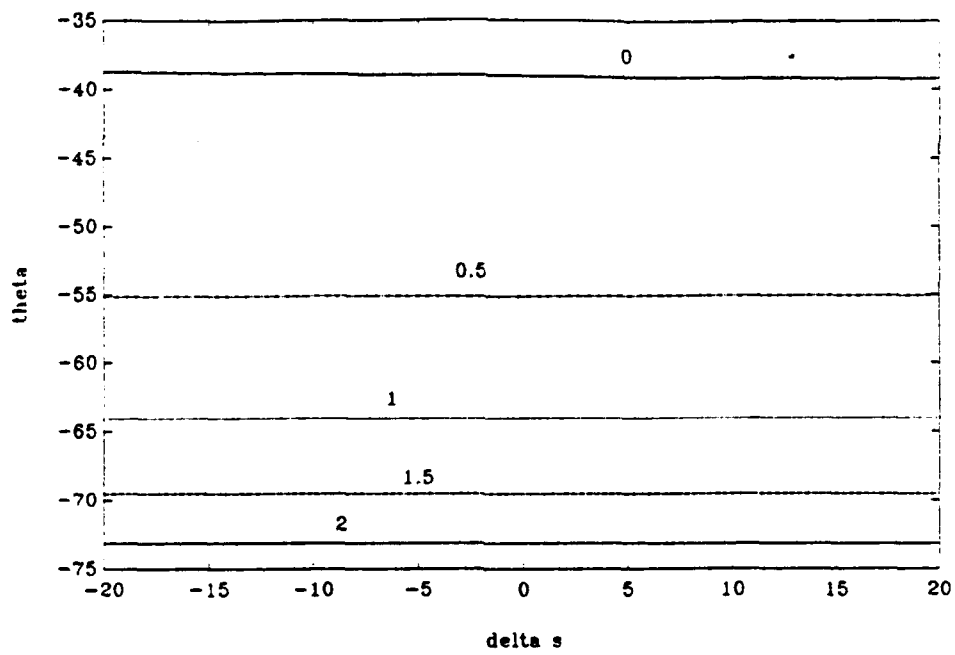


(a) $x_{GB} = 0, 0.5, 1.0, 1.5$ and 2.0%

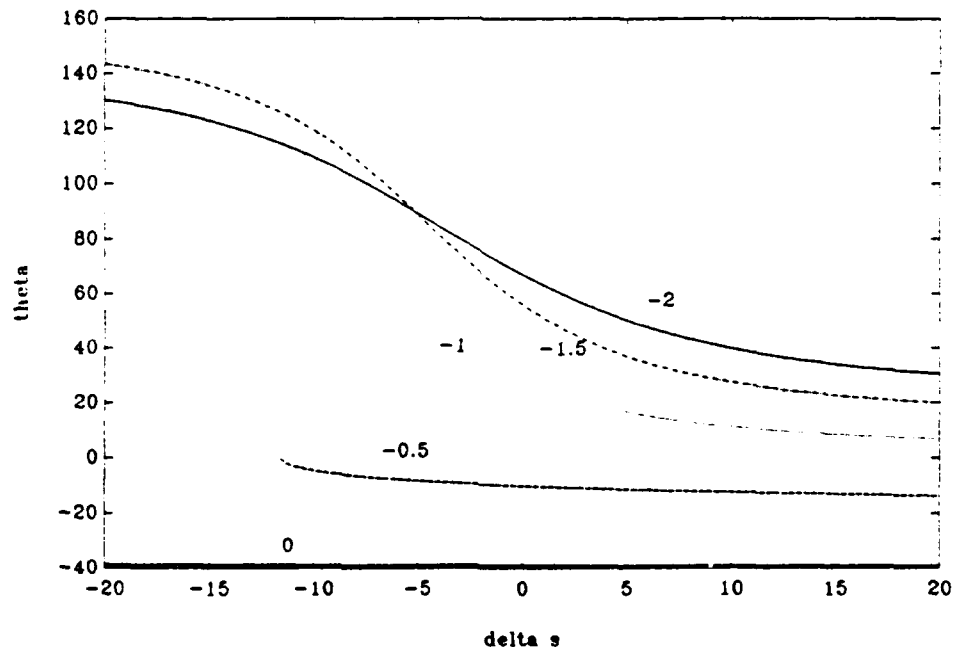


(b) $x_{GB} = 0, -0.5, -1.0, -1.5$ and -2.0%

Figure 13. Stable Heave Velocity (w) Solutions for Variations in x_{GB}

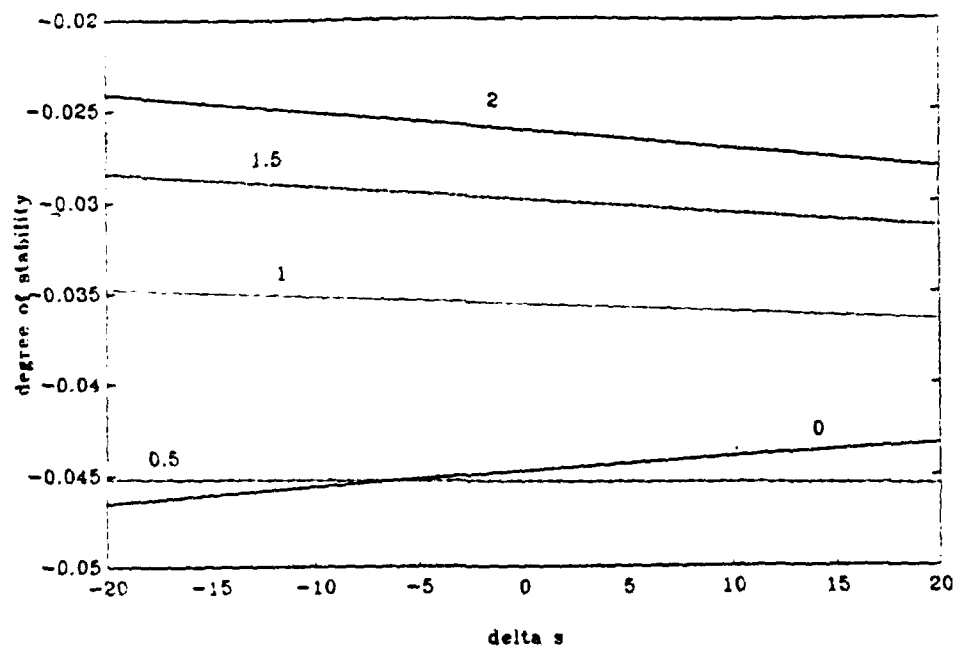


(a) $x_{GB} = 0, 0.5, 1.0, 1.5$ and 2.0 %

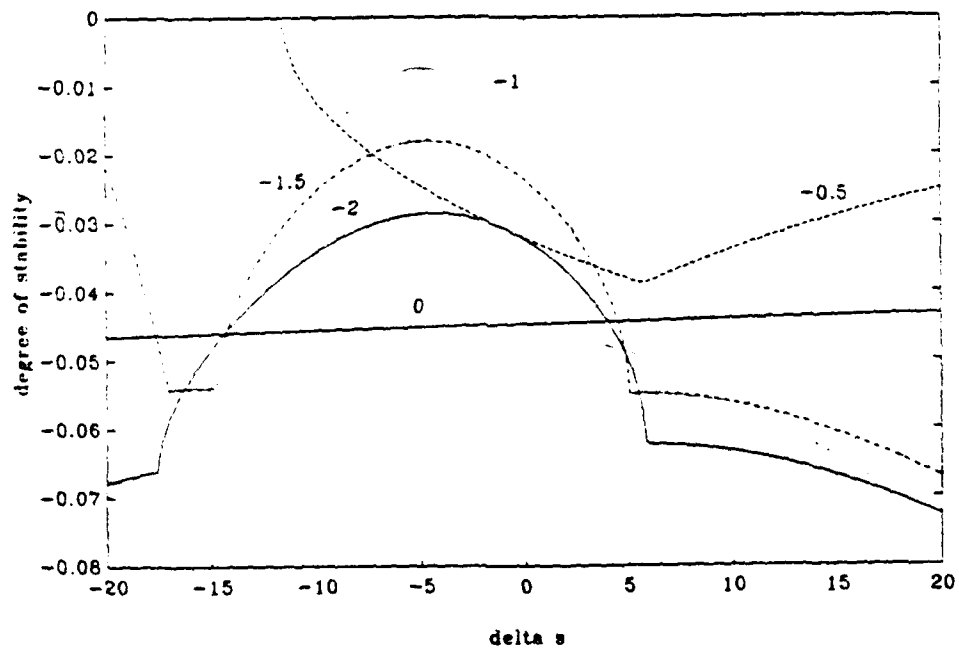


(b) $x_{GB} = 0, -0.5, -1.0, -1.5$ and -2.0 %

Figure 14. Stable Angle of Pitch (θ) Solutions for Variations in x_{GB}



(a) $x_{GB} = 0, 0.5, 1.0, 1.5$ and 2.0%

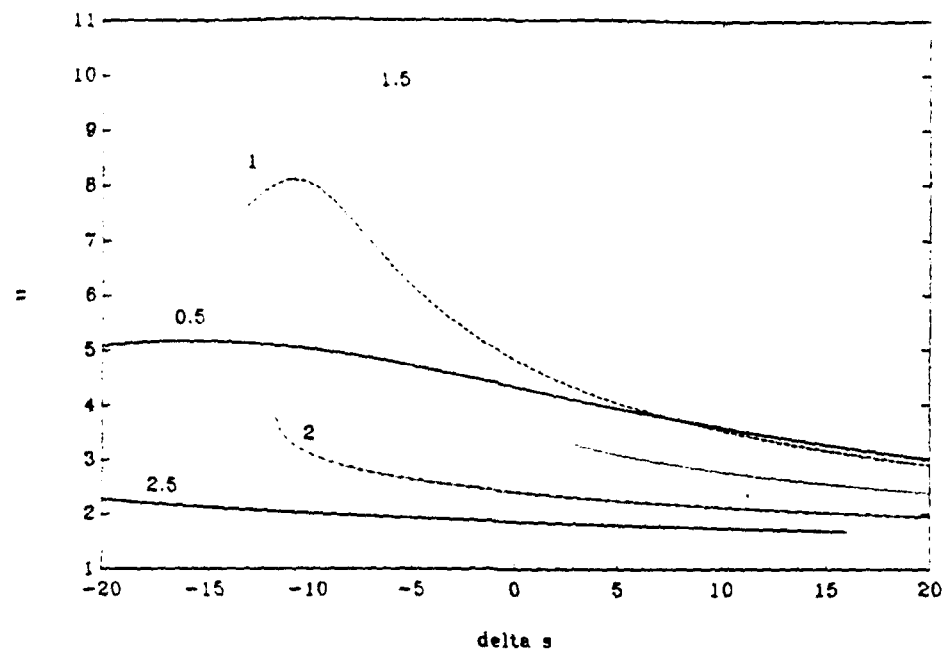


(b) $x_{GB} = 0, -0.5, -1.0, -1.5$ and -2.0%

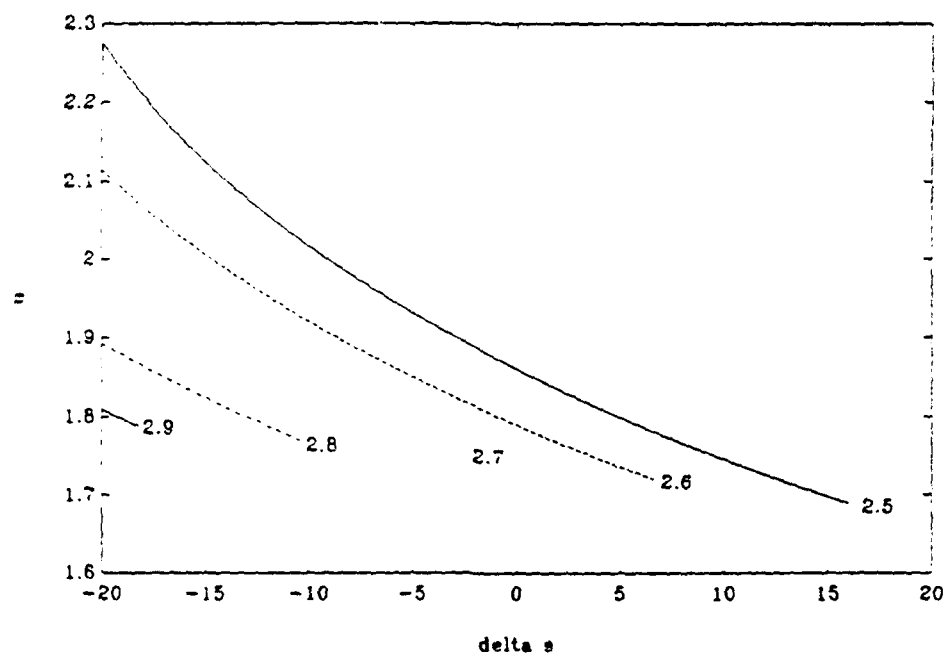
Figure 15. Degree of Stability for Variations in x_{GB}

2. Variations in the Amount of Excess Buoyancy (δB)

Figures 16 through 19 show how changing the amount of excess buoyancy (δB) effects the dynamic response of the submersible. For these cases, the bow plane deflection angle (δ_b) is zero, the longitudinal center of gravity (x_{GB}) is - 0.5 percent of vehicle length, the vertical center of gravity (z_{GB}) is 0.1 feet, and the longitudinal and vertical centers of buoyancy (x_B and z_B) are zero. The amount of excess buoyancy (δB) is varied from 0.5 to 2.9 percent of weight (W). The only case where there are stable solutions for the full range of dive plane angles is when buoyancy (δB) is 0.5.

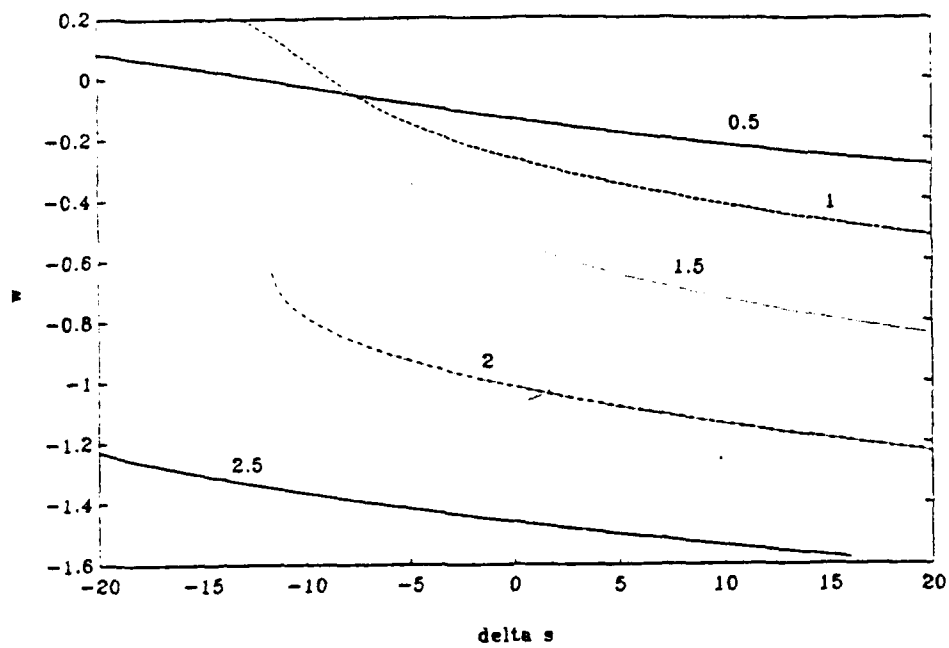


(a) $\delta B = 0.5, 1.0, 1.5$ and 2.0%

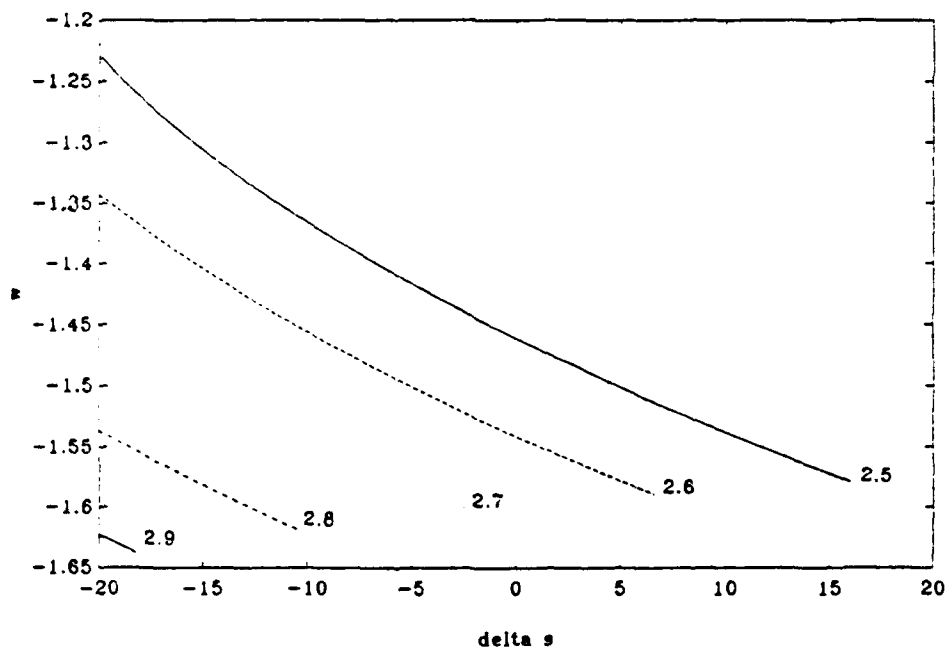


(b) $\delta B = 2.5, 2.6, 2.7, 2.8, 2.9 \%$

Figure 16. Stable Surge Velocity (u) Solutions for Variations in δB

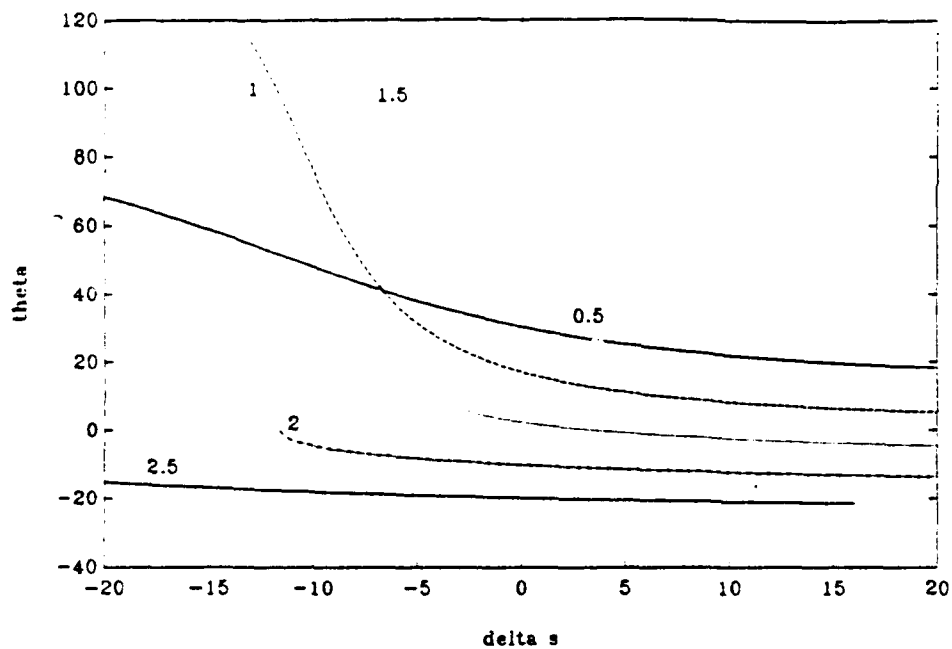


(a) $\delta B = 0.5, 1.0, 1.5$ and 2.0 %

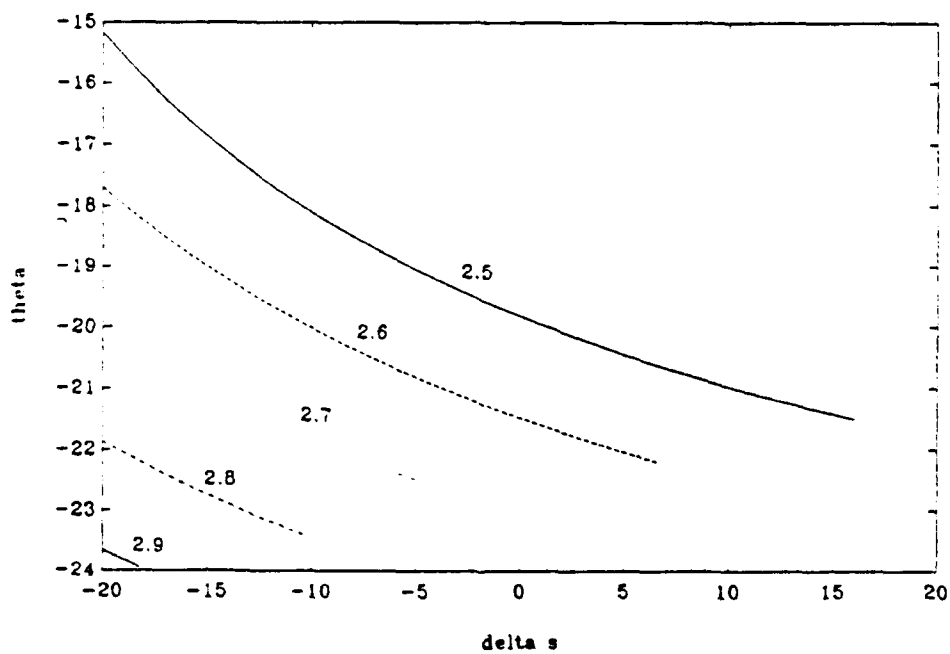


(b) $\delta B = 2.5, 2.6, 2.7, 2.8, 2.9$ %

Figure 17. Stable Heave Velocity (w) Solutions for Variations in δB

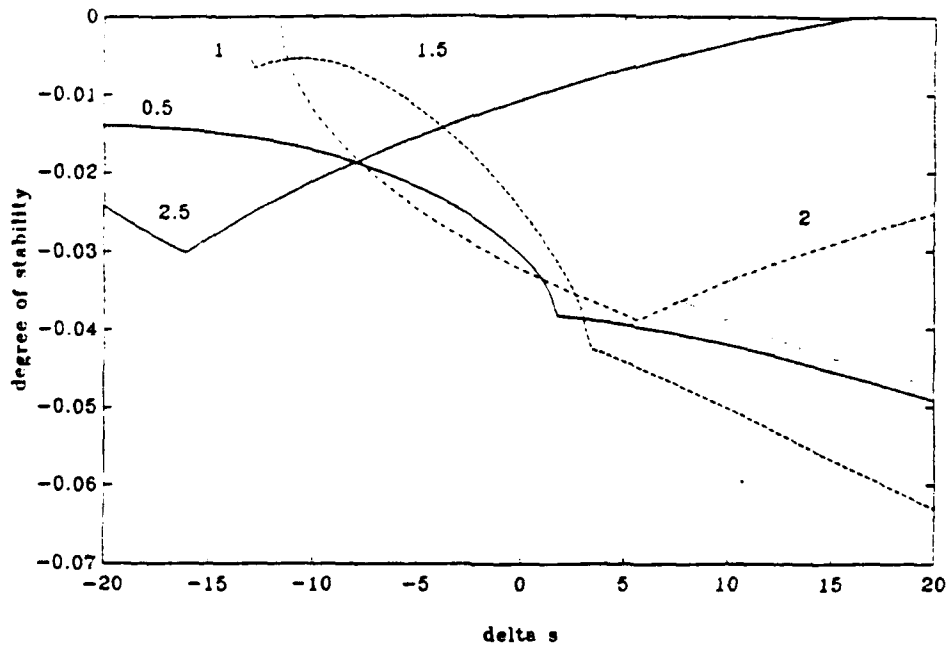


(a) $\delta B = 0.5, 1.0, 1.5$ and 2.0%

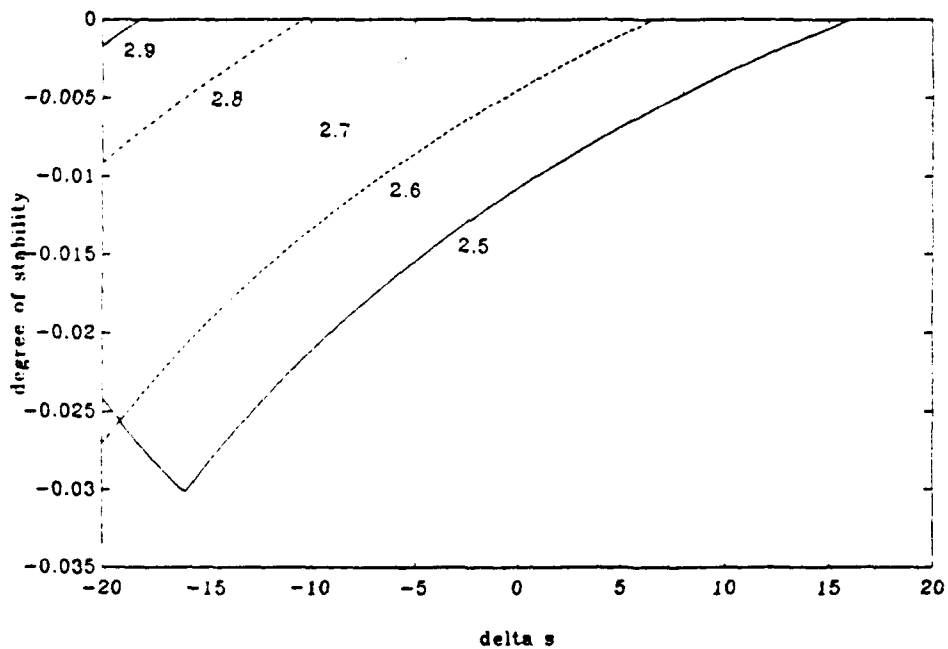


(b) $\delta B = 2.5, 2.6, 2.7, 2.8, 2.9 \%$

Figure 18. Stable Angle of Pitch (θ) Solutions for Variations in δB



(a) $\delta B = 0.5, 1.0, 1.5$ and 2.0%



(b) $\delta B = 2.5, 2.6, 2.7, 2.8, 2.9 \%$

Figure 19. Degree of Stability for Variations in δB

3. Variations in Vertical Center of Gravity (z_{GB})

Figures 20 through 23 show how changing the vertical center of gravity (z_{GB}) effects the dynamic response of the submersible. For these cases, the amount of excess buoyancy (δB) is two percent of weight (W), the bow plane deflection angle (δ_b) is zero, the longitudinal center of gravity (x_{GB}) is - 0.5 percent of vehicle length, and the longitudinal and vertical centers of buoyancy (x_B and z_B) are zero. The vertical center of gravity (z_{GB}) is varied from 0.05 to 0.30 feet. The general trend shown in these figures is that the larger values for vertical center of gravity (z_{GB}) have stable solutions over a larger range of dive plane angles.

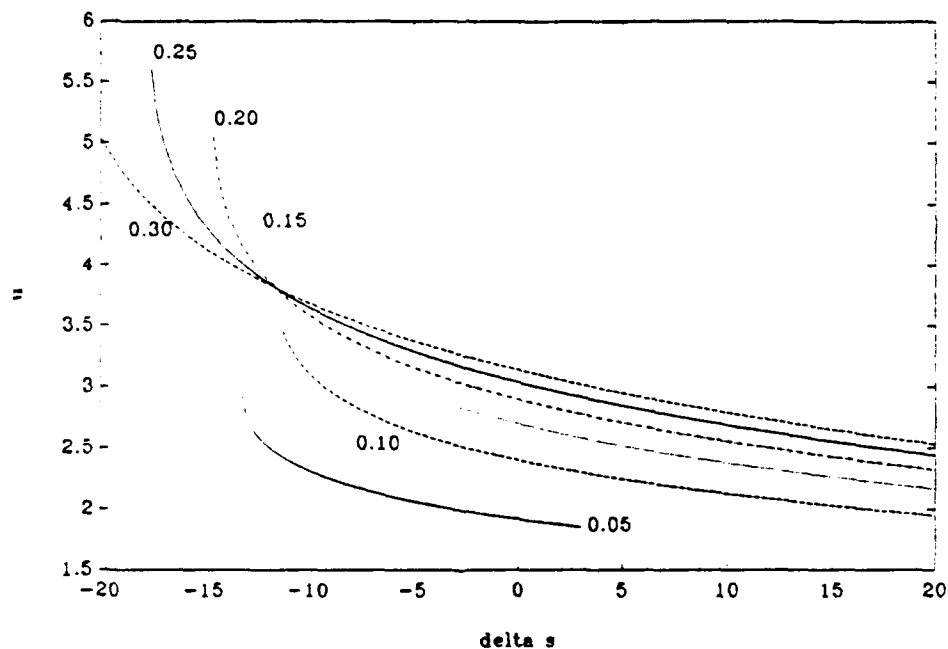


Figure 20. Stable Surge Velocity (u) Solutions for Variations in z_{GB}

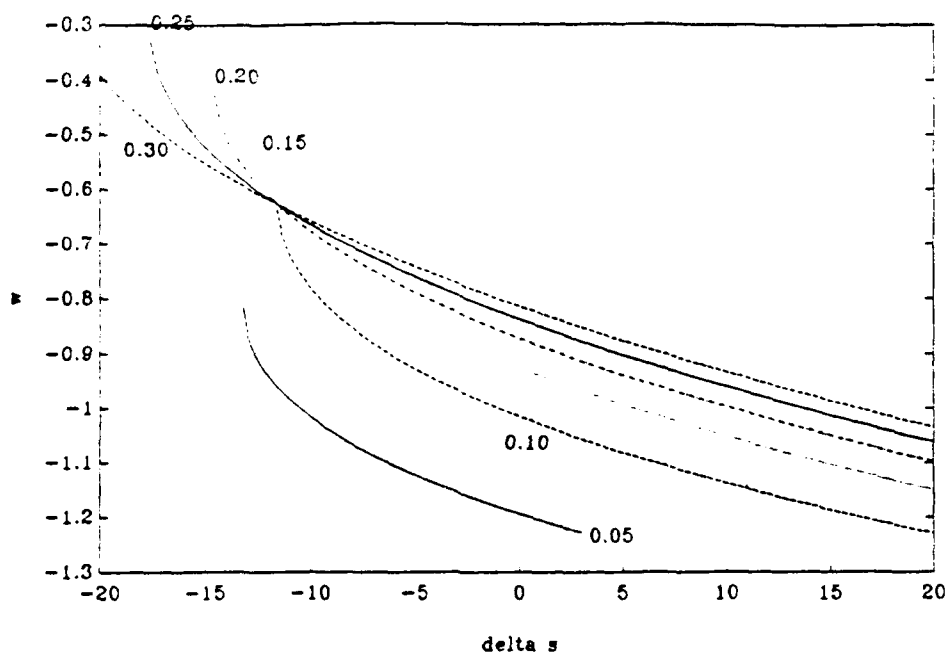


Figure 21. Stable Heave Velocity (w) Solutions for Variations in z_{GB}

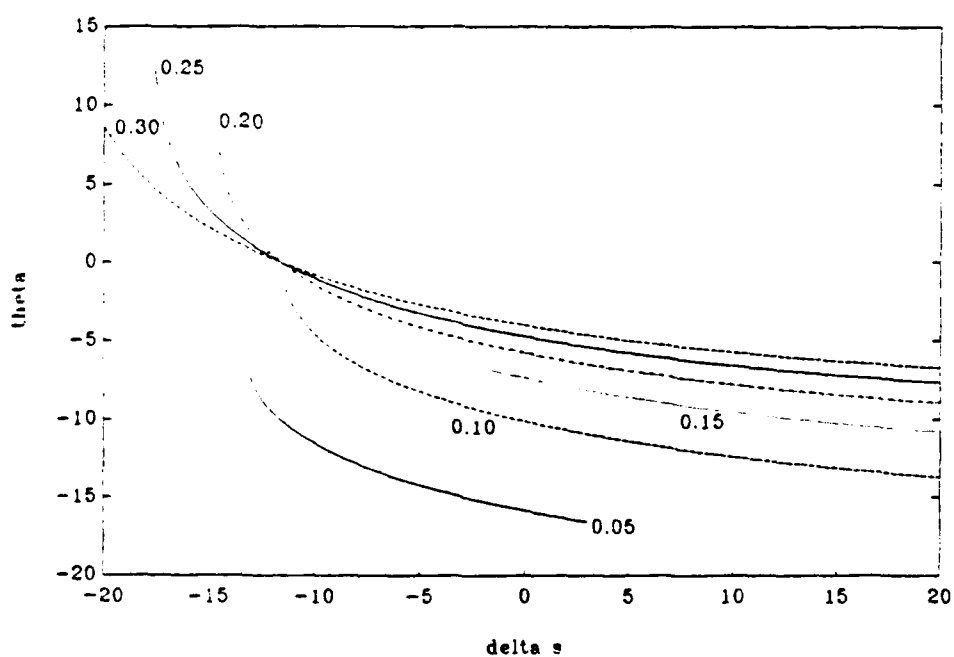


Figure 22. Stable Angle of Pitch (θ) Solutions for Variations in z_{GB}

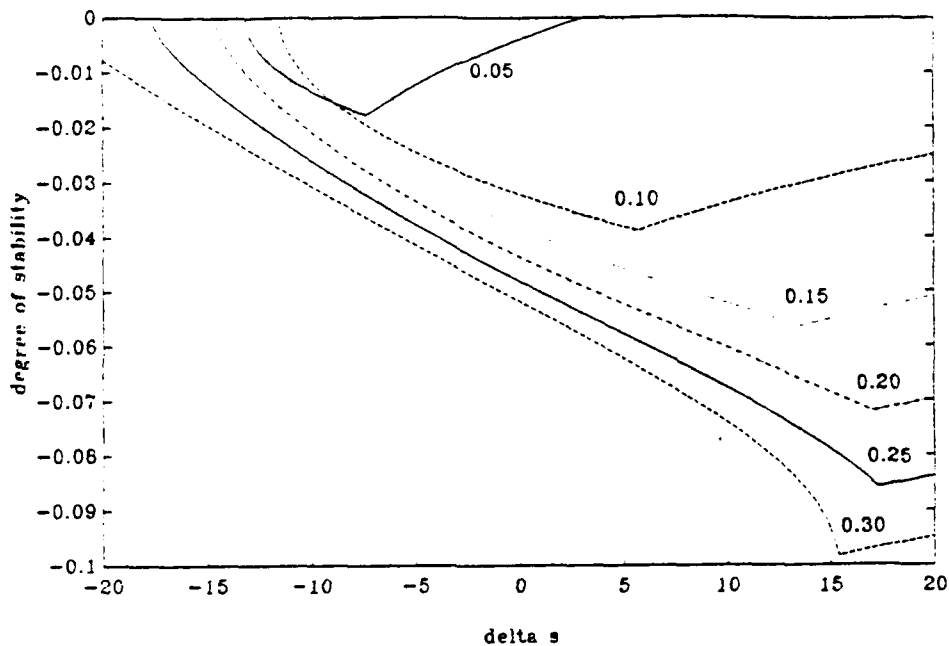


Figure 23. Degree of Stability for Variations in z_{GB}

4. Variations in the Longitudinal Center of Buoyancy (x_B)

Figures 24 through 27 show how changing the longitudinal center of buoyancy (x_B) effects the dynamic response of the submersible. For these cases, the amount of excess buoyancy (δB) is two percent of weight (W), the bow plane deflection angle (δ_b) is zero, the longitudinal center of gravity (x_{GB}) is - 0.5 percent of vehicle length, the vertical center of gravity (z_{GB}) is 0.1 feet, and the vertical center of buoyancy (z_B) is zero. The longitudinal center of buoyancy (x_B) is varied from - 9 to + 2 percent of vehicle length. The general trend shown in these figures is that the positive values for longitudinal center of buoyancy (x_B) tend to have stable solutions for positive dive plane angles. On the other hand, the negative values for longitudinal center of buoyancy (x_B) tend to have stable solutions for negative dive plane angles.

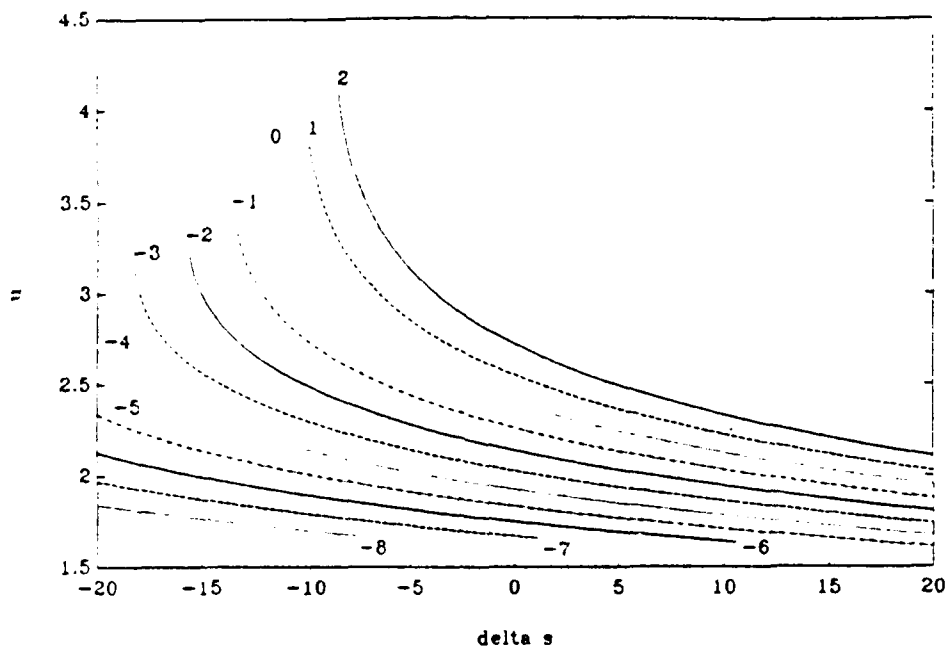


Figure 24. Stable Surge Velocity (u) Solutions for Variations in x_B

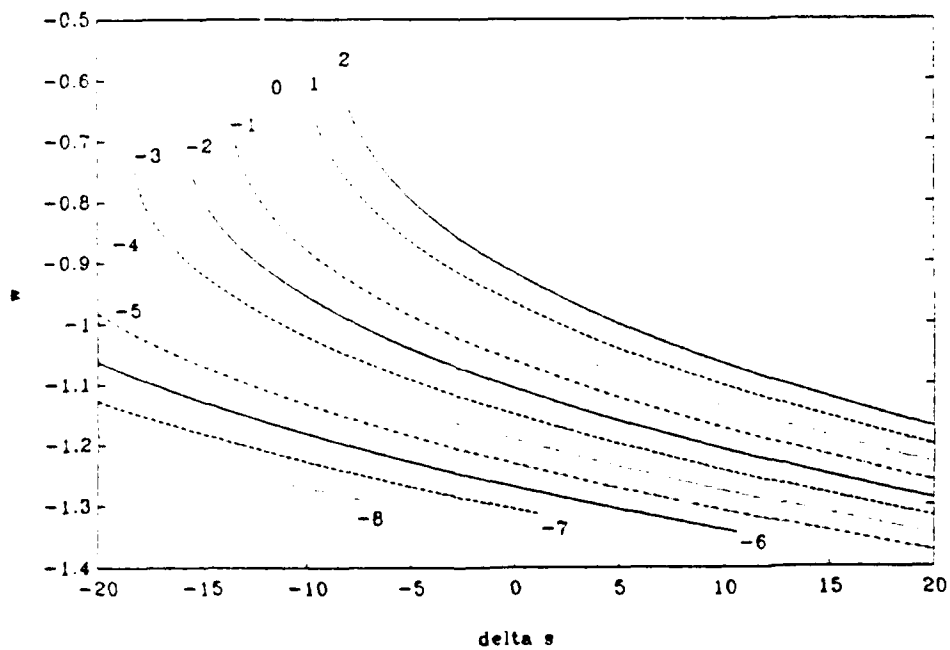


Figure 25. Stable Heave Velocity (w) Solutions for Variations in x_B

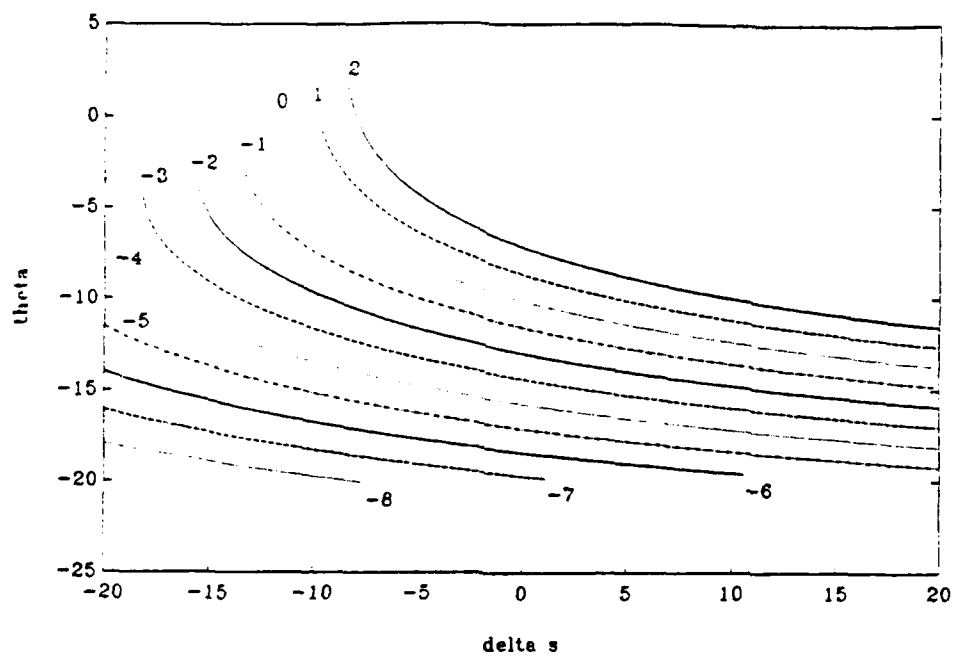


Figure 26. Stable Angle of Pitch (θ) Solutions for Variations in x_B

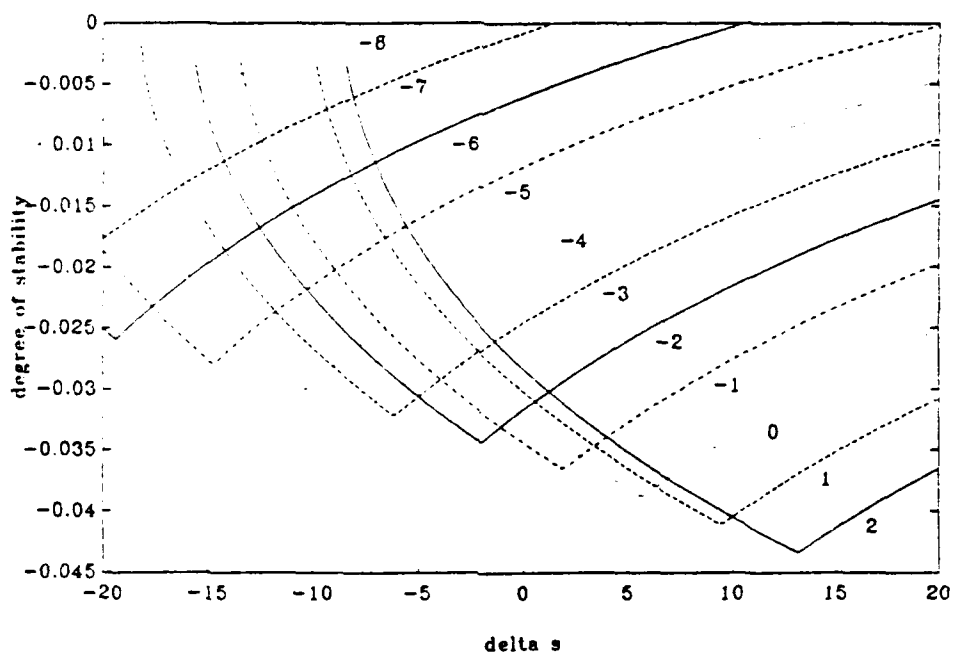


Figure 27. Degree of Stability for Variations in x_B

5. Variations in Bow Planes Deflection Angle (δ_b)

Figures 28 through 31 show how a non-zero bow planes deflection angle (δ_b) effects the dynamic response of the submersible. For these cases, the amount of excess buoyancy (δB) is two percent of weight (W), the longitudinal center of gravity (x_{GB}) is - 0.5 percent of vehicle length, the vertical center of gravity (z_{GB}) is 0.1 feet, and the longitudinal and vertical centers of buoyancy (x_B and z_B) are zero. The bow planes are given a deflection of - 20 degrees. The significance of the results shown in these figures is that for certain dive plane angles ($\delta_s = -3$ to -12 degrees) there are two stable solutions.

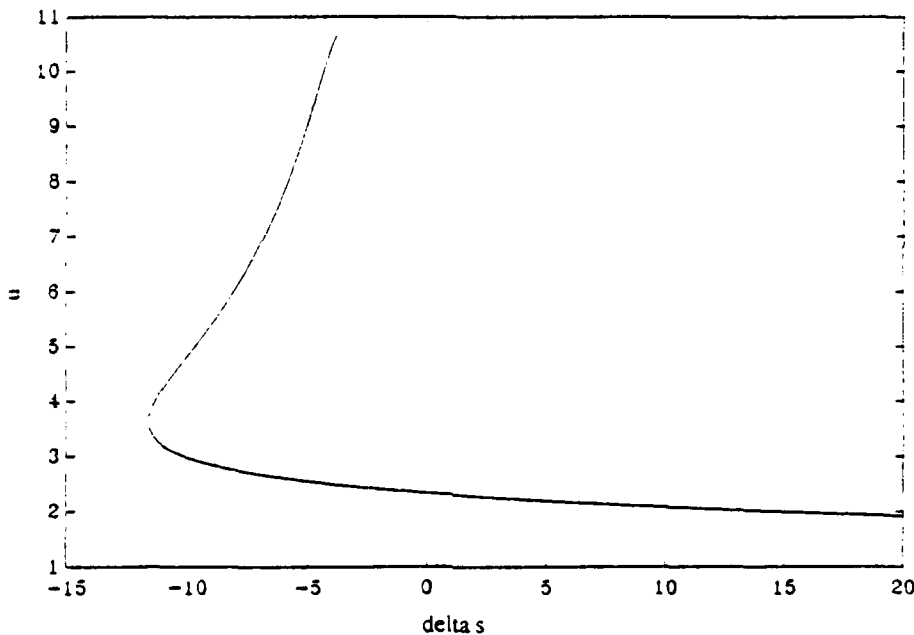


Figure 28. Stable Surge Velocity (u) Solutions for a Non-zero Bow Plane Angle ($\delta_b = -20$ degrees)

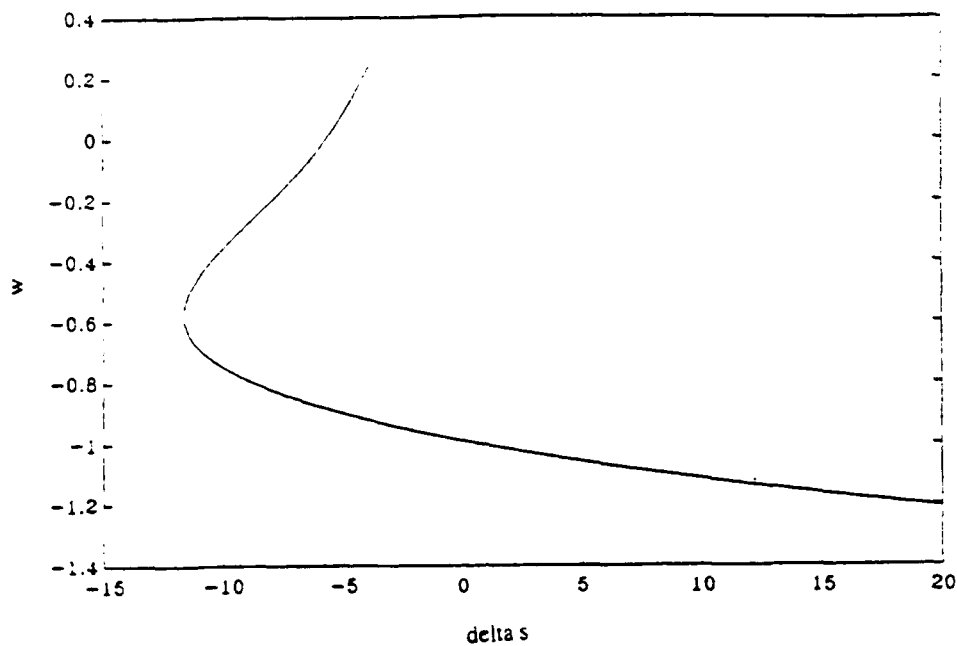


Figure 29. Stable Heave Velocity (w) Solutions for a Non-zero Bow Plane Angle ($\delta_b = -20$ degrees)

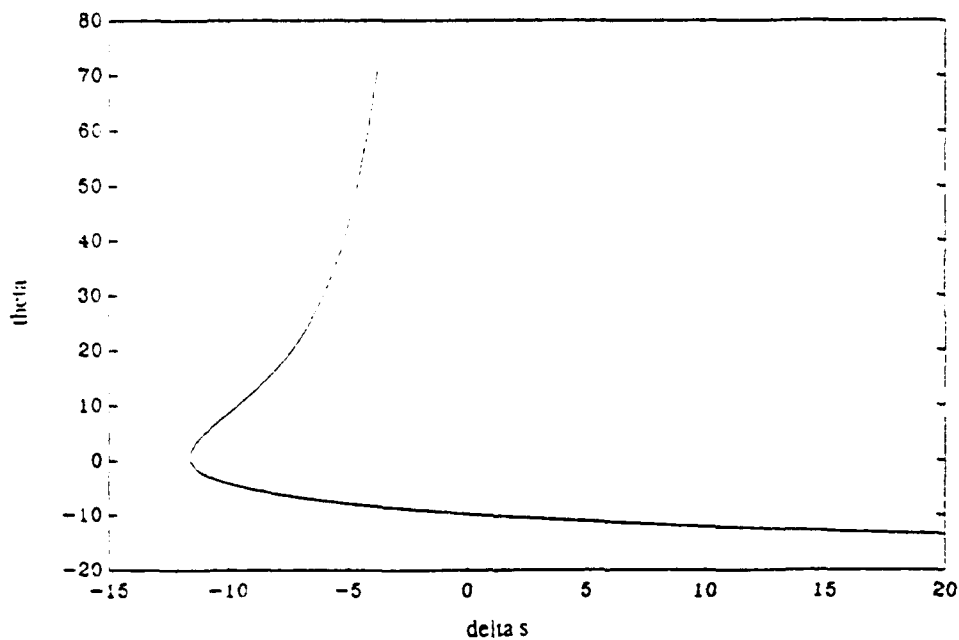


Figure 30. Stable Pitch Angle (θ) Solutions for a Non-zero Bow Plane Angle ($\delta_b = -20$ degrees)

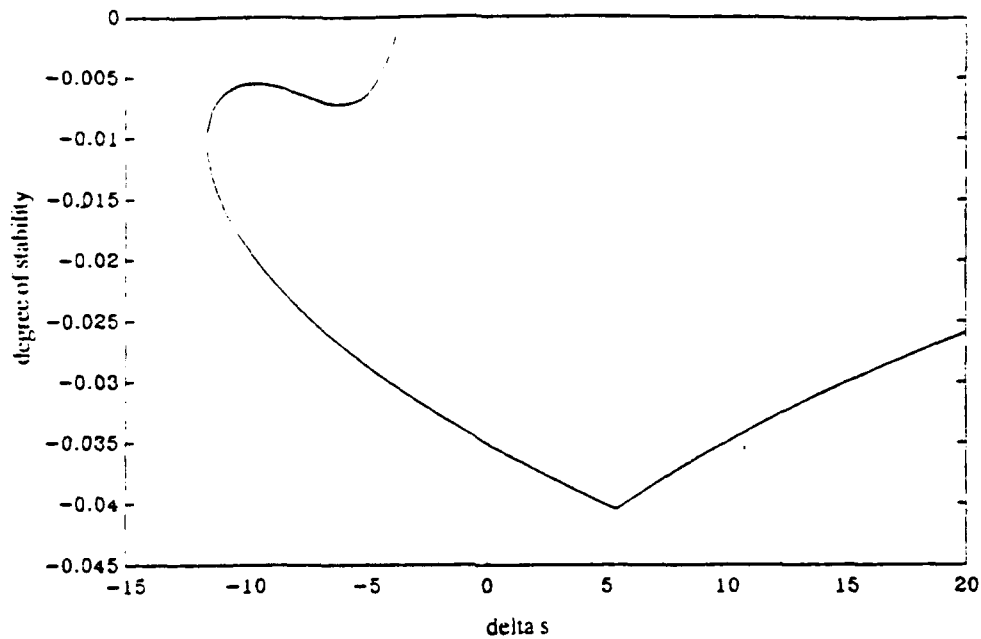


Figure 31. Degree of Stability for a Non-zero Bow Plane Angle
 $(\delta_b = -20 \text{ degrees})$

B. SIMULATIONS USING NUMERICAL INTEGRATION METHODS

The linearized dynamic response results were verified by simulations using numerical integration of the full six degrees of freedom equations of motion for the swimmer delivery vehicle (SDV). Figure 32 shows a plot of angle of pitch (θ) versus time for the center of gravity forward of the center of buoyancy case ($x_{GB} = +1$) with a dive plane angle (δ_s) of -15 degrees. The dotted line shows the linearized results from figure 7, the solid line shows the numerical integration results. The steady state results of the numerical integration method match the linearized dynamic results exactly.

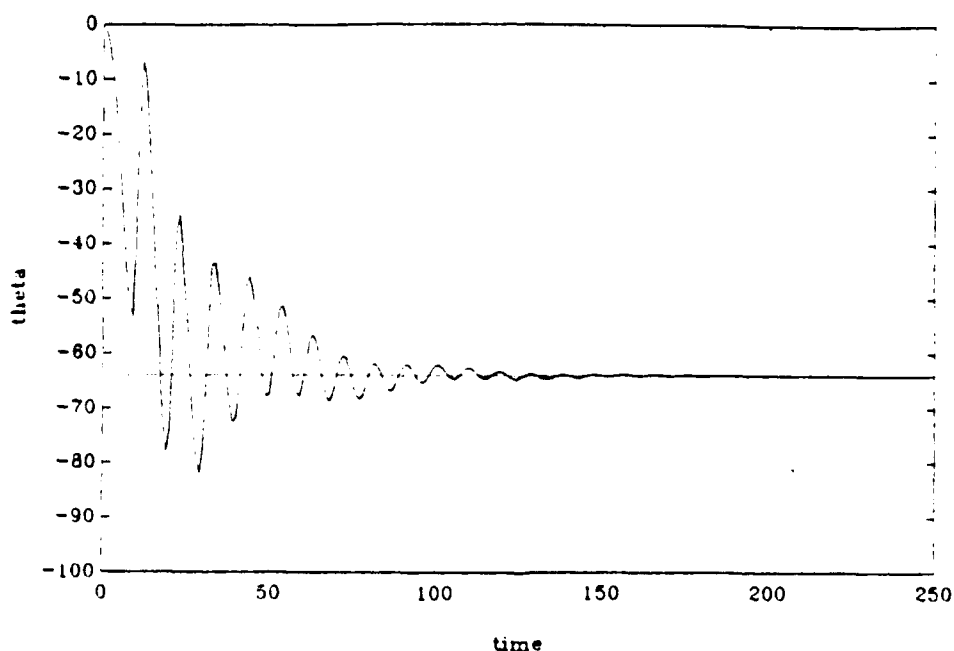


Figure 32. Numerical Integration Solution for Angle of Pitch (θ) when Center of Gravity is Forward ($x_{GB} = + 1\%$)

Figure 33 shows a plot of angle of pitch (θ) versus time for the center of gravity aft of the center of buoyancy case ($x_{GB} = - 1$) with a dive plane angle (δ_s) of $- 15$ degrees. Again the dotted line shows the linearized results from figure 7, the solid line shows the numerical integration results. And once again, the steady state results of the numerical integration method match the linearized dynamic results exactly. However, this linearized dynamic result was for the vertical plane only, the horizontal plane stability analysis indicated that this would be an unstable solution (figure 9). The reason for this disagreement in the results is investigated by adding an initial angle of roll to the numerical integration analysis. Adding a small angle of initial roll ($\phi_0 = 1$ degree) caused the vehicle to steady out at 137 degrees vice 159 degrees as shown in figure 34. This initial roll angle also caused a steady state roll angle of 17 degrees as shown in figure 35. In turn, this steady state roll angle caused the steady state yaw velocity (r) shown in figure 36 (i.e.

motion is no longer restricted to the vertical plane). Figure 37 shows a plot of z versus x and indicates that the vehicle is taking a helical ascent as discussed by Booth [Ref 2: 304-305]. Therefore, the numerical integration solution resulting in a steady state pitch angle of 159 degrees (figure 33) is unstable in the horizontal plane as predicted by the linearized dynamic response analysis.

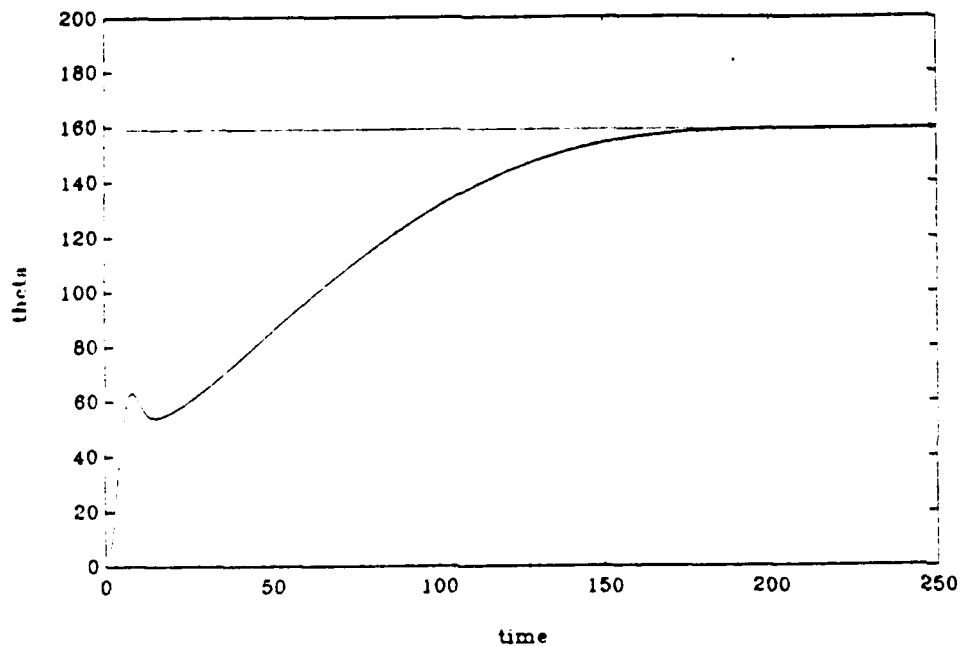


Figure 33. Numerical Integration Solution for Angle of Pitch (θ) when
Center of Gravity is Aft ($x_{GB} = -1\%$)

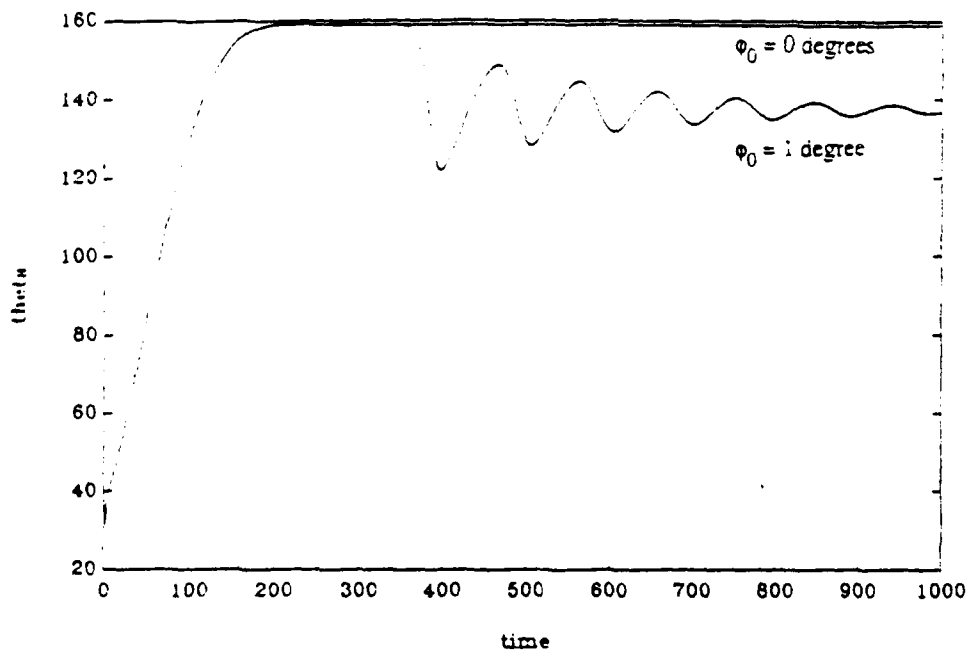


Figure 34. Numerical Integration Solution for Angle of Pitch (θ) when $x_{GB} = -1\%$ and Initial Roll Angle (ϕ_0) is 1 degree

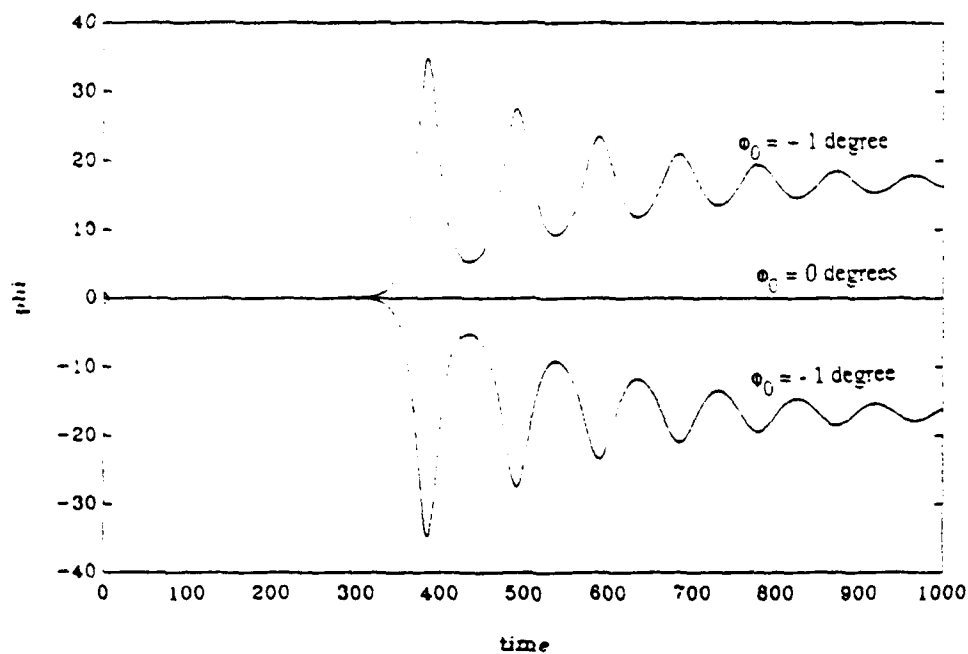


Figure 35. Numerical Integration Solution for Angle of Roll (ϕ) when $x_{GB} = -1\%$ and Initial Roll Angle (ϕ_0) is 1 degree

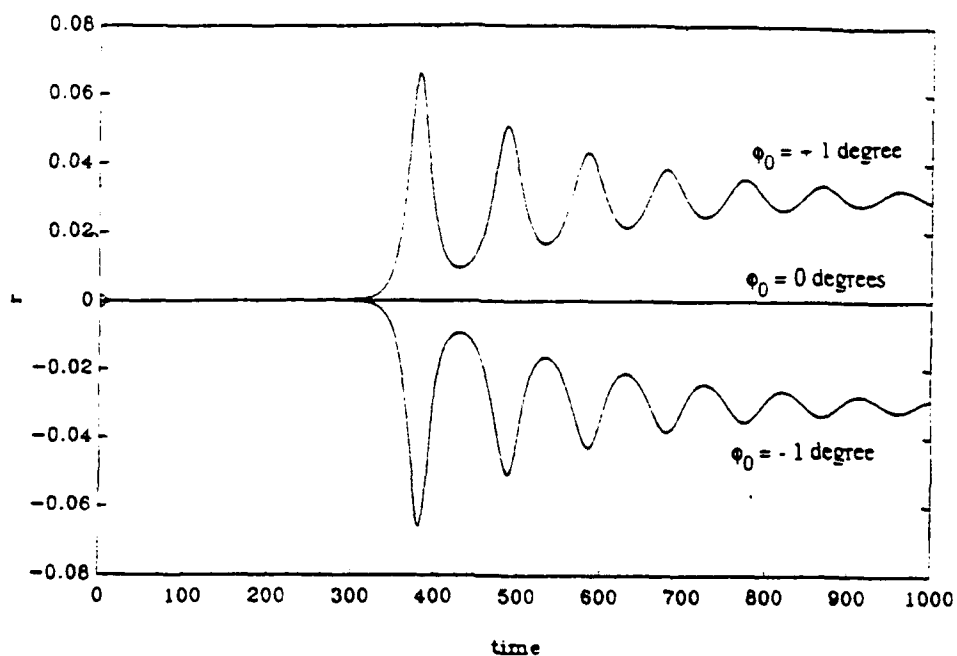


Figure 36. Numerical Integration Solution for Yaw Velocity (r) when $x_{GB} = -1\%$ and Initial Roll Angle (ϕ_0) is 1 degree

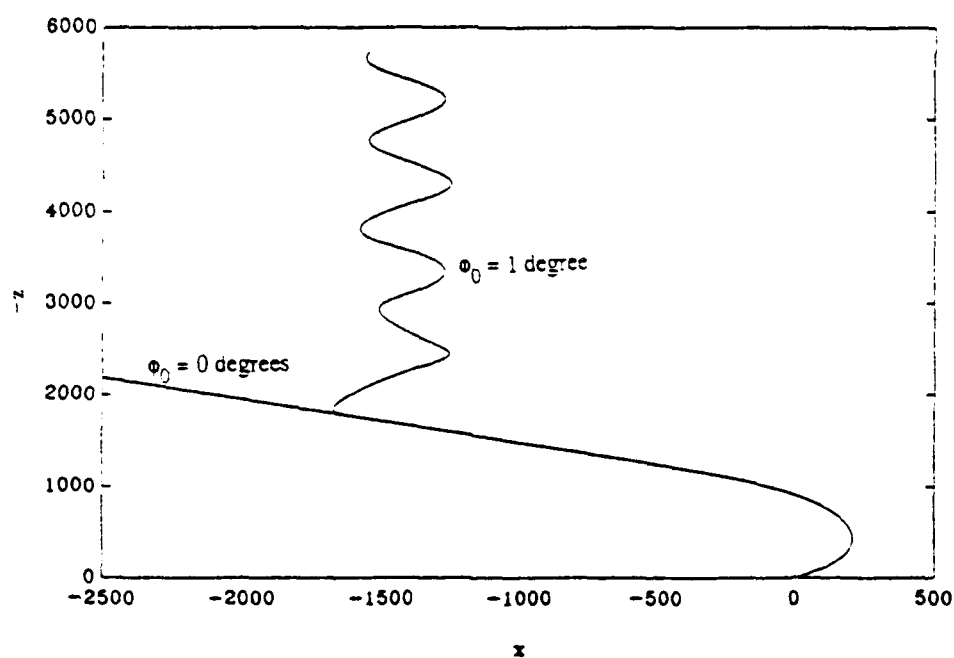


Figure 37. Numerical Integration Solution for Heave Velocity (z) when $x_{GB} = -1\%$ and Initial Roll Angle (ϕ_0) is 1 degree

Some question still remains with regards to the measure of stability of the 'inverted pendulum' solutions predicted by both the linearized dynamic response analysis and the numerical integration analysis. The linearized dynamic response analysis predicts a stable solution for the case when center of gravity is placed aft of center of buoyancy ($x_{GB} = -1\%$) and the dive plane angle (δ_s) is -7 degrees (figure 10). The corresponding steady state value for pitch angle was 118 degrees (figure 7). A random persistent roll disturbance (ϕ_d) was added to the numerical integration model and the results are shown in figure 38. The solid line indicates the results when a small disturbance is added (ϕ_d centered about 0.1 degrees), the dashed line indicates the results when a large disturbance is added (ϕ_d centered about 1.0 degrees). As expected, the large disturbance caused the vehicle to roll over as shown by the resulting angles of roll (ϕ) in figure 39. However, the vehicle continued to remain stable during small constant random disturbances in the inverted position as shown by the resulting angles of roll (ϕ) in figure 40. This indicates that indeed these 'inverted pendulum' solutions have a significant measure of stability.

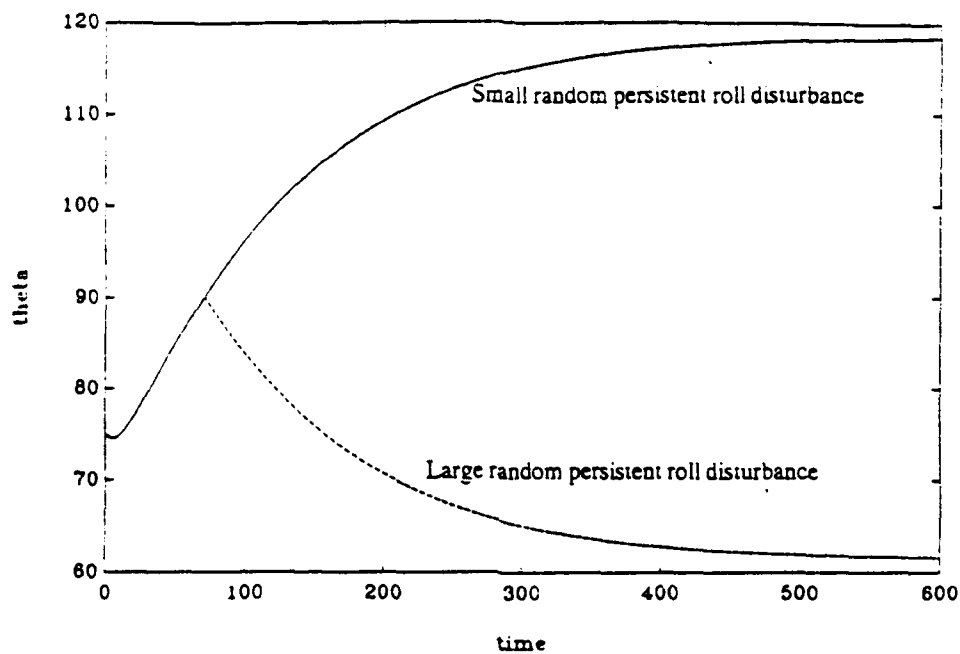


Figure 38. Numerical Integration Solution for Angle of Pitch (θ) when $x_{GB} = -1\%$ and a Persistent Roll Disturbance is Added

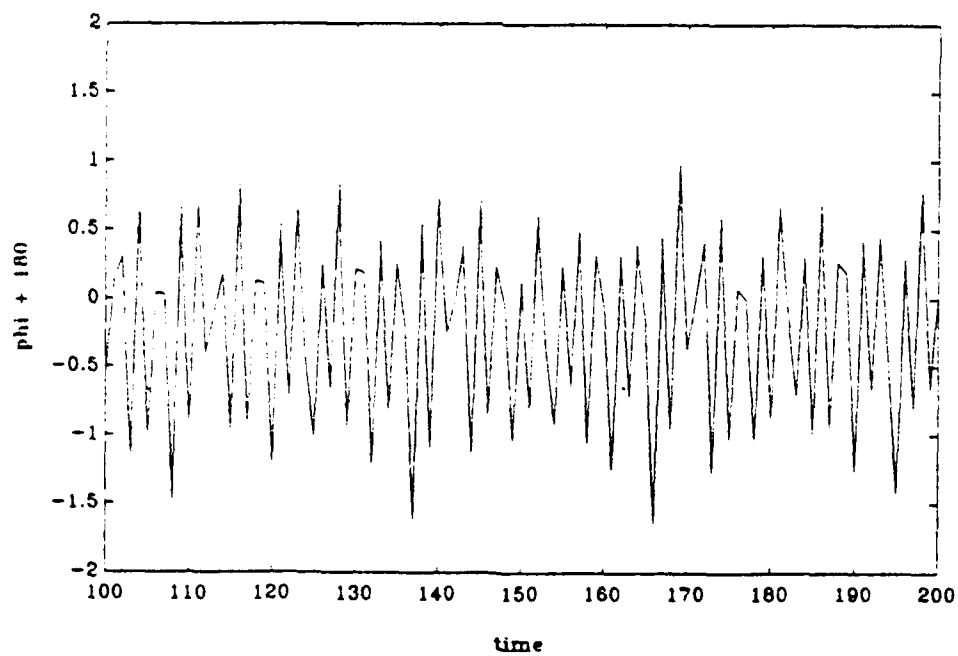


Figure 39. Numerical Integration Solution for Angle of Roll (ϕ) when $x_{GB} = -1\%$ and a Large Persistent Roll Disturbance is Added

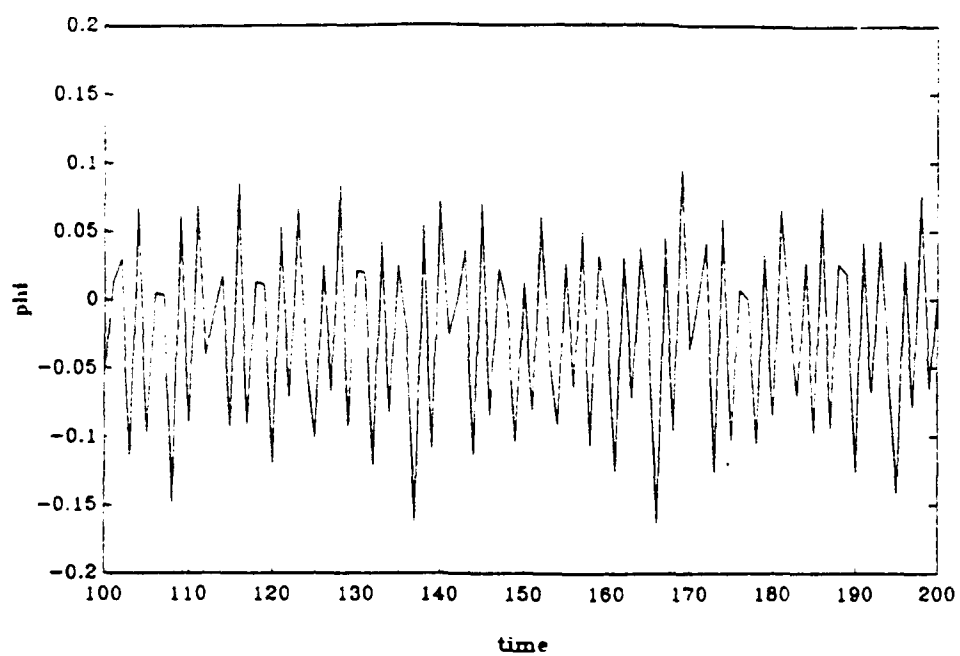


Figure 40. Numerical Integration Solution for Angle of Roll (ϕ) when $x_{GB} = -1\%$ and a Small Persistent Roll Disturbance is Added

VI. CONCLUSIONS AND RECOMMENDATIONS.

- The steady state analysis resulted in four possible solutions provided that vehicle motion was restricted to the vertical plane. Analyzing the dynamic stability using the steady state results as nominal points generally indicates that only one (if any) of the four possible solutions will be stable. There are a few cases where two solutions are stable, but these cases (certain non-zero bow plane deflection angles) appear to be the exception and not the rule.
- The dynamic stability characteristics of submersibles can be separated with respect to vertical plane motions (u, w, q, θ) and horizontal plane motions (v, p, r, ϕ).
- It is possible for submersibles to be dynamically stable with respect to vertical plane motion in the inverted (belly up) position during ascents ('Inverted Pendulum' stabilization).
- 'Inverted Pendulum' stabilization is also possible in the horizontal plane.
- Submersibles are able to maintain this inverted orientation (i.e. ascend belly up without rolling over) even under some persistent roll excitation.
- As a recommendation, the dynamic stability analysis should be expanded to include the case where angle of roll is 180 degrees, and the case where angle of roll is non-zero (i.e. ϕ neither equals zero nor 180 degrees). Analyzing the $\phi = 180$ degrees cases will only involve changing a few signs with regards to trigonometric functions; however, analyzing the non-zero cases will require significant effort.
- Furthermore, identifying and characterizing different stability regions over a range of variations of the system parameters should be the matter of future research.

APPENDIX A

SIX DEGREE OF FREEDOM EQUATIONS OF MOTION

Source: Smith, Crane, and Summey [Reference 1:11-16]

1. LONGITUDINAL (SURGE) EQUATION OF MOTION

$$\begin{aligned}
 m \left[\ddot{u} - v\dot{r} + w\dot{q} - x_G (q^2 + r^2) + y_G (pq - \dot{r}) + z_G (pr + \dot{q}) \right] = \\
 X_{pp} p^2 + X_{qq} q^2 + X_{rr} r^2 + X_{pr} pr \\
 + X_{\dot{u}} \dot{u} + X_{wq} wq + X_{vp} vp + X_{vr} vr \\
 + uq (X_{q\delta s} \delta_s + X_{q\delta b} \delta_b) + X_{r\delta r} ur\delta_r \\
 + X_{vv} v^2 + X_{ww} w^2 + X_{v\delta r} uv\delta_r + uw (X_{w\delta s} \delta_s + X_{w\delta b} \delta_b) \\
 + u^2 (X_{\delta s\delta s} \delta_s^2 + X_{\delta b\delta b} \delta_b^2 + X_{\delta r\delta r} \delta_r^2) - (W - B) \sin \theta \\
 + u^2 X_{prop}
 \end{aligned}$$

2. LATERAL (SWAY) EQUATION OF MOTION

$$\begin{aligned}
 m \left[\ddot{v} + ur - w\dot{p} + x_G (pq + \dot{r}) - y_G (p^2 + r^2) + z_G (qr - \dot{p}) \right] = \\
 Y_{pp} p^2 + Y_{rr} r^2 + Y_{pq} pq + Y_{qr} qr \\
 + Y_{\dot{v}} \dot{v} + Y_{p\dot{u}} p\dot{u} + Y_{r\dot{u}} r\dot{u} + Y_{vq} vq + Y_{wp} wp + Y_{wr} wr \\
 + Y_{v\dot{u}} v\dot{u} + Y_{vw} vw + Y_{\delta r} u^2 \delta_r \\
 - \int_{x_{tail}}^{x_{nose}} C_{Dy} h(x) (v+xr)^2 + C_{Dz} b(x) (w-xq)^2 \cdot \frac{(v+xr)}{U_{cf}(x)} dx \\
 + (W-B) \cos \theta \sin \phi
 \end{aligned}$$

3. NORMAL (HEAVE) EQUATION OF MOTION

$$\begin{aligned}
 m [\ddot{w} - u\dot{q} + v\dot{p} + x_G (\dot{p}\dot{r} - \dot{q}) + y_G (\dot{q}\dot{r} + \dot{p}) - z_G (\dot{p}^2 + \dot{q}^2)] = \\
 [Z_{\dot{q}\dot{q}} + Z_{pp} \dot{p}^2 + Z_{pr} \dot{p}\dot{r} + Z_{rr} \dot{r}^2] \\
 + [Z_{\dot{w}} \dot{w} + Z_{qu} u\dot{q} + Z_{vp} v\dot{p} + Z_{vr} v\dot{r}] \\
 + [Z_{uw} u\dot{w} + Z_{vv} v^2 + u^2 (Z_{\delta s} \delta_s + Z_{\delta b} \delta_b)] \\
 - \int_{x_{tail}}^{x_{nose}} [C_{Dy} h(x) (v + x\dot{r})^2 + C_{Dz} b(x) (w - x\dot{q})^2] \frac{(w - x\dot{q})}{U_{cf}(x)} dx \\
 + (W - B) \cos \theta \cos \phi
 \end{aligned}$$

4. ROLL EQUATION OF MOTION

$$\begin{aligned}
 I_x \ddot{p} + (I_z - I_y) \dot{q}\dot{r} + I_{xy} (\dot{p}\dot{r} - \dot{q}) - I_{yz} (\dot{q}^2 - \dot{r}^2) \\
 - I_{xz} (\dot{p}\dot{q} + \dot{r}) + m [y_G (\ddot{w} - u\dot{q} + v\dot{p}) - z_G (\ddot{v} + u\dot{r} - w\dot{p})] = \\
 [K_{\dot{p}} \dot{p} + K_{\dot{r}} \dot{r} + K_{pq} \dot{p}\dot{q} + K_{qr} \dot{q}\dot{r}] \\
 + [K_{\dot{v}} \dot{v} + K_p u\dot{p} + K_r u\dot{r} + K_{vq} v\dot{q} + K_{wp} w\dot{p} + K_{wr} w\dot{r}] \\
 + [K_v uv + K_{vw} vw] + (y_G W - y_B B) \cos \theta \cos \phi \\
 - (z_G W - z_B B) \cos \theta \sin \phi + u^2 K_{prop}
 \end{aligned}$$

5. PITCH EQUATION OF MOTION

$$\begin{aligned}
 & I_y \dot{q} + (I_x - I_y) pr - I_{xy} (qr + \dot{p}) + I_{yz} (pq - \dot{r}) \\
 & + I_{xz} (p^2 - r^2) - m [x_G (\dot{w} - uq + vp) - z_G (\dot{u} - vr + wp)] = \\
 & [M_q \dot{q} + M_{pp} p^2 + M_{pr} pr + M_{rr} r^2] \\
 & + [M_{\dot{w}} \dot{w} + M_q uq + M_{vp} vp + M_{vr} vr] \\
 & + [M_w uw + M_{vv} v^2 + u^2 (M_{\delta_s} \delta_s + M_{\delta_b} \delta_b)] \\
 & + \int_{x_{tail}}^{x_{nose}} \left[C_{Dy} h(x) (v+xr)^2 + C_{Dz} b(x) (w-xq)^2 \right] \frac{(w-xq)}{U_{cf}(x)} x dx \\
 & - (x_G W - x_B B) \cos \theta \cos \phi - (z_G W - z_B B) \sin \theta
 \end{aligned}$$

6. YAW EQUATION OF MOTION

$$\begin{aligned}
 & I_z \dot{r} + (I_y - I_x) pq - I_{xy} (p^2 - \dot{q}^2) - I_{yz} (pr + \dot{q}) \\
 & + I_{xz} (qr - \dot{p}) - m [x_G (\dot{v} + ur - wp) - y_G (\dot{u} - vr + wq)] = \\
 & [N_p \dot{p} + N_r \dot{r} + N_{pq} pq + N_{qr} qr] \\
 & + [N_{\dot{v}} \dot{v} + N_p up + N_r ur + N_{vq} vq + N_{wp} wp + N_{wr} wr] \\
 & + [N_v uv + N_{vw} vw + N_{\delta_r} u^2 \delta_r] \\
 & - \int_{x_{tail}}^{x_{nose}} \left[C_{Dy} h(x) (v+xr)^2 + C_{Dz} b(x) (w-xq)^2 \right] \frac{(v+xr)}{U_{cf}(x)} x dx \\
 & + (x_G W - x_B B) \cos \theta \sin \phi + (y_G W - y_B B) \sin \theta + u^2 N_{prop}
 \end{aligned}$$

APPENDIX B

ROTATION SEQUENCE AND EULER ANGLE RATES

1. ROTATION SEQUENCE FOR ϕ , θ AND ψ

Smith, Crane, and Summey [Reference 1:8] describe the transition from body fixed to inertial reference frames as follows:

Since the equations of motion are referred to an axis system that is fixed for the SDV (swimmer delivery vehicle), and thus translates and rotates with it, the orientation and position of the moving body axis system relative to a fixed inertial reference system must be specified. The orientation of the body axis system with respect to the inertial reference system is defined by the standard Euler angles ψ (yaw), θ (pitch), and ϕ (roll). The rotation sequence from the inertial reference system to the body system is ψ , θ , and ϕ as shown in Figure B1 taken from Smith, Crane, and Summey [Reference 1:18].

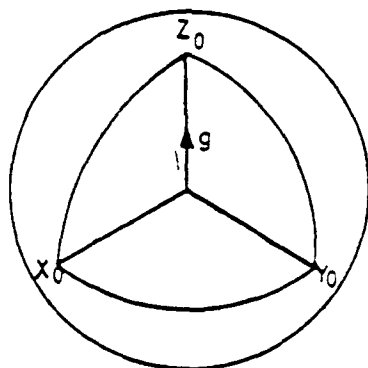
2. EULER ANGLE RATES FOR ϕ , θ AND ψ

The Euler angle rates used along with the six equations of motion (Appendix A) in order to completely determine the motion of the submersible were specified by Smith, Crane, and Summey [Reference 1:20] to be:

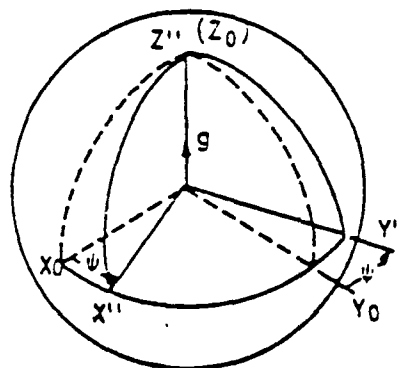
$$\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

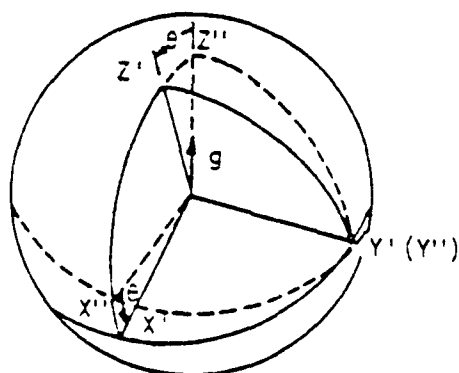
$$\dot{\psi} = q \frac{\sin \phi}{\cos \theta} + \frac{\cos \phi}{\cos \theta} r$$



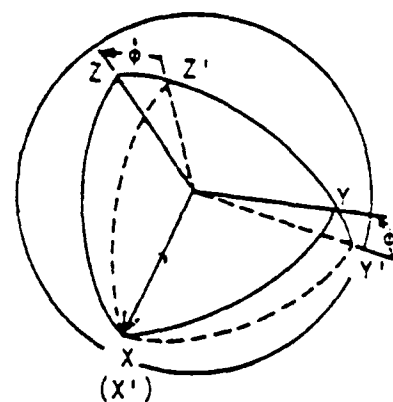
(1) Vehicle-Centered Gravity-Directed System parallel to inertial axis system.



(2) Horizontal Flight Reference System derived from $X_0Y_0Z_0$ by rotation about Z through yaw angle ψ .



(3) Roll-Resolved Flight Reference System derived from $X''Y''Z''$ by rotation about Y'' through pitch angle θ .



(4) Vehicle Body Axis Reference System derived from $X'Y'Z'$ by rotation about X' through roll angle ϕ .

Figure B1. Unit Sphere Development of Euler Angles

APPENDIX C

STEADY STATE COMPUTER PROGRAM

```
C  PROGRAM  STEADY
C  STEADY STATE SOLUTIONS IN THE VERTICAL PLANE
C  DIVE PLANE VARIATION
C
REAL L,MASS,IX,IY,IZ,IXZ,IYZ,IXY,LAMBDA
REAL KPDOT,KRDOT,KPQ,KQR,KV DOT,KP,KR,KVQ,KWP,KWR,KV
REAL KVW,KPN,KDB
REAL MQDOT,MPP,MPR,MRR,MW DOT,MQ,MVP,MVR,MW,MVV,MDS
REAL MDB,NDRB
REAL NPDOT,NRDOT,NPQ,NQR,NV DOT,NP,NR,NVQ,NWP,NWR,NV
REAL NVW,NDRS
C
DIMENSION X(9),BR(9),HH(9),VECL(9)
C
C  GEOMETRIC PROPERTIES AND HYDRODYNAMIC COEFFICIENTS
C
PI      =4.0*ATAN(1.0)
WEIGHT=12000.0
L       = 17.425
RHO     = 1.94
G       = 32.2
CD0     = 0.0057
MASS    =WEIGHT/G
CDZ     =0.5*0.5*RHO
XWW     = 1.710E-01*0.5*RHO*L**2
XWDS    = 4.600E-02*0.5*RHO*L**2
XWDB    = 9.660E-03*0.5*RHO*L**2
XDSDS   =-1.160E-02*0.5*RHO*L**2
XDBDB   =-8.070E-03*0.5*RHO*L**2
CD0     = CD0*0.5*RHO*L**2
ZW      =-3.020E-01*0.5*RHO*L**2
ZDS     =-2.270E-02*0.5*RHO*L**2
ZDB     =-2.270E-02*0.5*RHO*L**2
MW       = 9.860E-02*0.5*RHO*L**3
MDS     =-1.113E-02*0.5*RHO*L**3
MDB     = 1.113E-02*0.5*RHO*L**3
C
OPEN(21,NAME='ST1.RES',STATUS='NEW')
OPEN(22,NAME='ST2.RES',STATUS='NEW')
OPEN(23,NAME='ST3.RES',STATUS='NEW')
OPEN(24,NAME='ST4.RES',STATUS='NEW')
OPEN(31,NAME='COEF.DAT',STATUS='NEW')
```


C DEFINE THE LENGTH X, BREADTH BR, AND HEIGHT HH TERMS
C

```
X(1) = -105.9/12.0
X(2) = -99.3/12.0
X(3) = -87.3/12.0
X(4) = -66.3/12.0
X(5) = 72.7/12.0
X(6) = 83.2/12.0
X(7) = 91.2/12.0
X(8) = 99.2/12.0
X(9) = 103.2/12.0
```

C

```
BR(1) = 0.00/12.0
BR(2) = 8.24/12.0
BR(3) = 19.76/12.0
BR(4) = 29.36/12.0
BR(5) = 31.85/12.0
BR(6) = 27.84/12.0
BR(7) = 21.44/12.0
BR(8) = 12.00/12.0
BR(9) = 0.00/12.0
```

C

C

COMPUTE AREA AND CENTROID

C

```
CALL TRAP(9,BR,X,AREA)
DO 9 I=1,9
  VEC1(I)=X(I)*BR(I)
9 CONTINUE
CALL TRAP(9,VEC1,X,XAA)
XA=XAA/AREA
```

C

```
WRITE (*,1002)
READ (*,*)        DSMIND,DSMAXD,IDS
DSMIN=DSMIND*PI/180
DSMAX=DSMAXD*PI/180
WRITE (*,1001)
READ (*,*)        RATIO
WRITE (*,1003)
READ (*,*)        DELB
DELB=DELB*WEIGHT/100.0
WRITE (*,1004)
READ (*,*)        XGB
XGB=XGB*L/100.0
WRITE (*,1005)
READ (*,*)        ZGB
WRITE (*,1006)
READ (*,*)        XB
XB=XB*L/100.0
WRITE (*,1007)
READ (*,*)        ZB
WRITE (31,*)      RATIO, DELB, XGB, ZGB, XB, ZB
```

C

```

DO 1 I=1,IDS
C
  DS=DSMIN+(DSMAX-DSMIN)*(I-1)/(IDS-1)
  IF (DELB.EQ.0.0) DELB=0.000001
  IF (ZGB.EQ.0.0) ZGB =0.000001
  DB=RATIO*DS
C
  PX      =XGB*WEIGHT-XB*DELB
  PZ      =ZGB*WEIGHT-ZB*DELB
  DEN     =CDZ*AREA*(PX+XA*DELB)
  LAMBDA=MW*DELB-PX*ZW+PZ*(XWDS+RATIO*XWDB)*DS
  ALPHA   =-PX*(ZDS+RATIO*ZDB)*DS-PZ*CD0+PZ*(XDSDS+
&          RATIO*RATIO*XDBDB)*DS**2+DELB*(MDS+RATIO
&          *MDB)*DS
  BETA    =PZ*XWW
  LAMBDA=LAMBDA/DEN
  ALPHA   =ALPHA /DEN
  BETA    =BETA  /DEN
C
C
  A  = 1.0+BETA
  B  = LAMBDA
  C  = ALPHA
  DET= B**2-4.0*A*C
  IF (DET.LT.0.0) GO TO 2
  WP=(-B+SQRT(DET))/(2.0*A)
  YY=-XWW*WP**2-(XWDS*DS+XWDB*DB)*WP
&      -(XDSDS*DS**2+XDBDB*DB**2)+CD0
  IF (WP.GE.0.0) XX=ZW*WP+ZDS*DS+ZDB*DB-CDZ*AREA
&      *WP*ABS(WP)
  IF (WP.LT.0.0) XX=ZW*WP+ZDS*DS+ZDB*DB+CDZ*AREA
&      *WP*ABS(WP)
  THETA=ATAN2(YY,XX)
  USQ=DELB*SIN(THETA)/YY
  THETA=THETA*180/PI
  DSD=DS*180/PI
  IF (USQ.LT.0.0) GO TO 3
  IF (WP.GE.0.0) U= SQRT(USQ)
  IF (WP.LT.0.0) U=-SQRT(USQ)
  W=WP*U
  WRITE (21,*) DSD,THETA,U,W,WP
C
3  WP=(-B-SQRT(DET))/(2.0*A)
  YY=-XWW*WP**2-(XWDS*DS+XWDB*DB)*WP
&      -(XDSDS*DS**2+XDBDB*DB**2)+CD0
  IF (WP.GE.0.0) XX=ZW*WP+ZDS*DS+ZDB*DB-CDZ*AREA
&      *WP*ABS(WP)
  IF (WP.LT.0.0) XX=ZW*WP+ZDS*DS+ZDB*DB+CDZ*AREA
&      *WP*ABS(WP)
  THETA=ATAN2(YY,XX)
  USQ=DELB*SIN(THETA)/YY

```

```

DSD=DS*180/PI
THETA=THETA*180/PI
IF (USQ.LT.0.0) GO TO 2
IF (WP.LT.0.0) U=-SQRT(USQ)
IF (WP.GE.0.0) U= SQRT(USQ)
W=WP*U
WRITE (22,*) DSD,THETA,U,W,WP

C
C
2  A  = -1.0+BETA
   DET= B**2-4.0*A*C
   IF (DET.LT.0.0) GO TO 1
   WP=(-B+SQRT(DET))/(2.0*A)
   YY=-XWW*WP**2-(XWDS*DS+XWDB*DB)*WP
   &    -(XDSDS*DS**2+XDBDB*DB**2)+CD0
   IF (WP.LT.0.0) XX=ZW*WP+ZDS*DS+ZDB*DB-CDZ*AREA
   *WP*ABS(WP)
   IF (WP.GE.0.0) XX=ZW*WP+ZDS*DS+ZDB*DB+CDZ*AREA
   *WP*ABS(WP)
   THETA=ATAN2(YY,XX)
   USQ=DELB*SIN(THETA)/YY
   DSD=DS*180/PI
   THETA=THETA*180/PI
   IF (USQ.LT.0.0) GO TO 4
   IF (WP.GE.0.0) U=-SQRT(USQ)
   IF (WP.LT.0.0) U= SQRT(USQ)
   W=WP*U
   WRITE (23,*) DSD,THETA,U,W,WP
4  WP=(-B-SQRT(DET))/(2.0*A)
   YY=-XWW*WP**2-(XWDS*DS+XWDB*DB)*WP
   &    -(XDSDS*DS**2+XDBDB*DB**2)+CD0
   IF (WP.LT.0.0) XX=ZW*WP+ZDS*DS+ZDB*DB-CDZ*AREA
   *WP*ABS(WP)
   IF (WP.GE.0.0) XX=ZW*WP+ZDS*DS+ZDB*DB+CDZ*AREA
   *WP*ABS(WP)
   THETA=ATAN2(YY,XX)
   USQ=DELB*SIN(THETA)/YY
   DSD=DS*180/PI
   THETA=THETA*180/PI
   IF (USQ.LT.0.0) GO TO 1
   IF (WP.GE.0.0) U=-SQRT(USQ)
   IF (WP.LT.0.0) U= SQRT(USQ)
   W=WP*U
   WRITE (24,*) DSD,THETA,U,W,WP

C
C
1  CONTINUE

C
STOP
1001 FORMAT (' ENTER BOW PLANE TO DIVE PLANE RATIO')
1002 FORMAT (' ENTER MIN, MAX, AND INCREMENTS IN
   &         DS (degrees)')
1003 FORMAT (' ENTER DELB (%W)')
1004 FORMAT (' ENTER XGB (%L)')

```

```

1005 FORMAT (' ENTER ZGB (feet)')
1006 FORMAT (' ENTER XB (%L)')
1007 FORMAT (' ENTER ZB (feet)')
      END

C      SUBROUTINE TRAP(N,A,B,OUT)
C
C      NUMERICAL INTEGRATION ROUTINE USING
C      THE TRAPEZOIDAL RULE
C
      DIMENSION A(1),B(1)
      N1=N-1
      OUT=0.0
      DO 1 I=1,N1
        OUT1=0.5*(A(I)+A(I+1))*(B(I+1)-B(I))
        OUT =OUT+OUT1
1 CONTINUE
      RETURN
      END

```

APPENDIX D

LINEARIZED DYNAMIC STABILITY COMPUTER PROGRAM

```

C      PROGRAM LINEARIZED DYNAMIC STABILITY
C      10      20      30      40      50
C234567890123456789012345678901234567890123456
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION L,MASS,IA,IX,IY,IZ
      DOUBLE PRECISION KPDOT,KRDOT,KPQ,KQR,KV DOT,KP,KR,
&   KVQ,KWP,KWR,KV,
&   KVW,KPN,KDB,MQDOT,MPP,MPR,MRR,MWDOT,MQ,MVP,MVR,
&   MW,MVV,MDS,MDB,
&   NDRB,NPDOT,NRDOT,NPQ,NQR,NV DOT,NP,NR,NVQ,NWP,NWR,
&   NV,NVW,NDRS
C
      DIMENSION A1(4,4),B1(4,4),BETA1(4),ALFR1(4),ALFI1(4)
      DIMENSION BB1(4,4),BB2(4,4),ZZZ1(4,4),ZZZ2(4,4)
      DIMENSION A2(4,4),B2(4,4),BETA2(4),ALFR2(4),ALFI2(4)
      DIMENSION WR1(4),WR2(4),WI1(4),WI2(4)
      DIMENSION X(9),BR(9),VEC1(9),VEC2(9)
C
C      GEOMETRIC PROPERTIES
C
      PI      = 4.D0*DATAN(1.D0)
      WEIGHT= 12000.0
      IX      = 1760.0
      IY      = 9450.0
      IZ      =10700.0
      L       = 17.425
      RHO     = 1.94
      G       = 32.2
      CD0     = 0.0057
      MASS    = WEIGHT/G
      CDZ     = 0.5*0.5*RHO
      CD0     = CD0*0.5*RHO*L**2
C
C      SURGE HYDRODYNAMIC COEFFICIENTS
C
      XPP     = 7.030E-03*0.5*RHO*L**4
      XQQ     =-1.470E-02*0.5*RHO*L**4
      XRR     = 4.010E-03*0.5*RHO*L**4
      XPR     = 7.640E-04*0.5*RHO*L**4
      XUDOT   =-7.580E-03*0.5*RHO*L**3
      XWQ     =-1.920E-01*0.5*RHO*L**3

```

XVP = -3.240E-03*0.5*RHO*L**3
 XVR = 1.890E-02*0.5*RHO*L**3
 XQDS = 2.610E-02*0.5*RHO*L**3
 XQDB = -2.600E-03*0.5*RHO*L**3
 XRDR = -8.180E-04*0.5*RHO*L**3
 XVV = 5.290E-02*0.5*RHO*L**2
 XWW = 1.710E-01*0.5*RHO*L**2
 XVDR = 1.730E-03*0.5*RHO*L**2
 XWDS = 4.600E-02*0.5*RHO*L**2
 XWDB = 9.660E-03*0.5*RHO*L**2
 XDSDS = -1.160E-02*0.5*RHO*L**2
 XDBDB = -8.070E-03*0.5*RHO*L**2
 XDRDR = -1.010E-02*0.5*RHO*L**2
 XRES = CD0*0.5*RHO*L**2

C
 C
 C

SWAY HYDRODYNAMIC COEFFICIENTS

YPDOT = 1.270E-04*0.5*RHO*L**4
 YRDOT = 1.240E-03*0.5*RHO*L**4
 YPQ = 4.125E-03*0.5*RHO*L**4
 YQR = -6.510E-03*0.5*RHO*L**4
 YVDOT = -5.550E-02*0.5*RHO*L**3
 YP = 3.055E-03*0.5*RHO*L**3
 YR = 2.970E-02*0.5*RHO*L**3
 YVQ = 2.360E-02*0.5*RHO*L**3
 YWP = 2.350E-01*0.5*RHO*L**3
 YWR = -1.880E-02*0.5*RHO*L**3
 YV = -9.310E-02*0.5*RHO*L**2
 YVW = 6.840E-02*0.5*RHO*L**2
 YDRS = +2.270E-02*0.5*RHO*L**2
 YDRB = +2.270E-02*0.5*RHO*L**2

C
 C
 C

HEAVE HYDRODYNAMIC COEFFICIENTS

ZQDOT = -6.810E-03*0.5*RHO*L**4
 ZPP = 1.270E-04*0.5*RHO*L**4
 ZPR = 6.670E-03*0.5*RHO*L**4
 ZRR = -7.350E-03*0.5*RHO*L**4
 ZWDOT = -2.430E-01*0.5*RHO*L**3
 ZQ = -1.350E-01*0.5*RHO*L**3
 ZVP = -4.810E-02*0.5*RHO*L**3
 ZVR = 4.550E-02*0.5*RHO*L**3
 ZW = -3.020E-01*0.5*RHO*L**2
 ZVV = -6.840E-02*0.5*RHO*L**2
 ZDS = -2.270E-02*0.5*RHO*L**2
 ZDB = -2.270E-02*0.5*RHO*L**2

C
 C
 C

ROLL HYDRODYNAMIC COEFFICIENTS

KPDOT = -1.010E-03*0.5*RHO*L**5
 KRDOT = -3.370E-05*0.5*RHO*L**5
 KPQ = -6.930E-05*0.5*RHO*L**5

```

KQR   = 1.680E-02*0.5*RHO*L**5
KVDOT = 1.270E-04*0.5*RHO*L**4
KP    = -1.100E-02*0.5*RHO*L**4
KR    = -8.410E-04*0.5*RHO*L**4
KVQ   = -5.115E-03*0.5*RHO*L**4
KWP   = -1.270E-04*0.5*RHO*L**4
KWR   = 1.390E-02*0.5*RHO*L**4
KV    = 3.055E-03*0.5*RHO*L**3
KVW   = -1.870E-01*0.5*RHO*L**3

```

C
C
C

PITCH HYDRODYNAMIC COEFFICIENT

```

MQDOT = -1.680E-02*0.5*RHO*L**5
MPP    = 5.260E-05*0.5*RHO*L**5
MPR    = 5.040E-03*0.5*RHO*L**5
MRR    = -2.860E-03*0.5*RHO*L**5
MWDOT = -6.810E-02*0.5*RHO*L**4
MQ     = -6.860E-02*0.5*RHO*L**4
MVP    = 1.180E-03*0.5*RHO*L**4
MVR    = 1.730E-02*0.5*RHO*L**4
MW     = 9.860E-02*0.5*RHO*L**3
MVV    = -2.510E-02*0.5*RHO*L**3
MDS    = -1.113E-02*0.5*RHO*L**3
MDB    = 1.113E-02*0.5*RHO*L**3

```

C
C
C

YAW HYDRODYNAMIC COEFFICIENTS

```

NPDOT = -3.370E-05*0.5*RHO*L**5
NRDOT = -3.400E-03*0.5*RHO*L**5
NPQ    = -2.110E-02*0.5*RHO*L**5
NQR    = 2.750E-03*0.5*RHO*L**5
NVDOT = 1.240E-03*0.5*RHO*L**4
NP     = -8.405E-04*0.5*RHO*L**4
NR     = -1.640E-02*0.5*RHO*L**4
NVQ    = -9.990E-03*0.5*RHO*L**4
NWP    = -1.750E-02*0.5*RHO*L**4
NWR    = 7.350E-03*0.5*RHO*L**4
NV     = -7.420E-03*0.5*RHO*L**3
NVW    = -2.670E-02*0.5*RHO*L**3
NDRS   = -1.113E-02*0.5*RHO*L**3
NDRB   = +1.113E-02*0.5*RHO*L**3

```

C
C
C

DEFINE THE LENGTH X AND BREADTH BR TERMS

```

X(1) = -105.9/12.0
X(2) = -99.3/12.0
X(3) = -87.3/12.0
X(4) = -66.3/12.0
X(5) = 72.7/12.0
X(6) = 83.2/12.0
X(7) = 91.2/12.0
X(8) = 99.2/12.0
X(9) = 103.2/12.0

```

```

C
BR(1) = 0.00/12.0
BR(2) = 8.24/12.0
BR(3) = 19.76/12.0
BR(4) = 29.36/12.0
BR(5) = 31.85/12.0
BR(6) = 27.84/12.0
BR(7) = 21.44/12.0
BR(8) = 12.00/12.0
BR(9) = 0.00/12.0

C
C
C
COMPUTE AREA, CENTROID, AND MOMENT OF INERTIA

CALL TRAP(9,BR,X,AREA)
DO 9 I=1,9
    VEC1(I)=X(I)*BR(I)
    VEC2(I)=X(I)*VEC1(I)
9 CONTINUE
CALL TRAP(9,VEC1,X,XAA)
XA=XAA/AREA
CALL TRAP(9,VEC2,X,IA)

C
WRITE (*,1001)
READ (*,*) IRES
OPEN(31,NAME='COEF.DAT',STATUS='OLD')
READ(31,*) RATIO, DELB, XGB, ZGB, XB, ZB
BUOY= WEIGHT + DELB
XG=XB+XGB
ZG=ZB+ZGB

C
C
C
MASS MATRIX COEFFICIENTS

B1(1,1)= MASS - XUDOT
B1(1,2)= 0.0
B1(1,3)= MASS*ZG
B1(1,4)= 0.0

C
B1(2,1)= 0.0
B1(2,2)= MASS - ZWDOT
B1(2,3)= -(ZQDOT+MASS*XG)
B1(2,4)= 0.0

C
B1(3,1)= MASS*ZG
B1(3,2)= -(MWDOT+MASS*XG)
B1(3,3)= IY-MQDOT
B1(3,4)= 0.0

C
B1(4,1)= 0.0
B1(4,2)= 0.0
B1(4,3)= 0.0
B1(4,4)= 1.0

C

```



```

      B2(1,1)= IX-KPDOT
      B2(1,2)= 0.0
      B2(1,3)=-(KVDOT+MASS*ZG)
      B2(1,4)=-KRDOT
C
      B2(2,1)= 0.0
      B2(2,2)= 1.0
      B2(2,3)= 0.0
      B2(2,4)= 0.0
C
      B2(3,1)=-(YPDOT+MASS*ZG)
      B2(3,2)= 0.0
      B2(3,3)= MASS-YVDOT
      B2(3,4)= MASS*XG-YRDOT
C
      B2(4,1)=-NPDOT
      B2(4,2)= 0.0
      B2(4,3)= MASS*XG-NVDOT
      B2(4,4)= IZ-NRDOT
C
      OPEN(41,NAME='DEOS.RES',STATUS='NEW')
      OPEN(42,NAME='DEOS1.RES',STATUS='NEW')
      OPEN(43,NAME='DEOS2.RES',STATUS='NEW')
C
      IF (IRES.EQ.1) GO TO 1
      IF (IRES.EQ.2) GO TO 2
      IF (IRES.EQ.3) GO TO 3
      IF (IRES.EQ.4) GO TO 4
1  OPEN (21,NAME='ST1.RES',STATUS='OLD')
11 READ (21,*,END=100) DSD,THET0,U0,W0,WP
   GO TO 5
2  OPEN (22,NAME='ST2.RES',STATUS='OLD')
12 READ (22,*,END=100) DSD,THET0,U0,W0,WP
   GO TO 5
3  OPEN (23,NAME='ST3.RES',STATUS='OLD')
13 READ (23,*,END=100) DSD,THET0,U0,W0,WP
   GO TO 5
4  OPEN (24,NAME='ST4.RES',STATUS='OLD')
14 READ (24,*,END=100) DSD,THET0,U0,W0,WP
   GO TO 5
C
5  THETA0=THET0*PI/180.0
   DS = DSD*PI/180.0
   DB = DS * RATIO
C
C  DAMPING MATRIX COEFFICIENTS
C
      A1(1,1)=-2.0*U0*CD0+W0*(XWDS*DS+XWDB*DB)
&      +2.0*U0*(XDSDS*DS**2+XDBDB*DB**2)
      A1(1,2)= 2.0*XWW*W0+U0*(XWDS*DS+XWDB*DB)
      A1(1,3)= (XWQ-MASS)*W0+(XQDS*DS+XQDB*DB)*U0
      A1(1,4)=-(WEIGHT-BUOY)*DCOS(THETA0)

```

```

C      A1(2,1)= ZW*W0+2.0*U0*(ZDS*DS+ZDB*DB)
      A1(2,2)= ZW*U0-2.0*CDZ*AREA*DABS(W0)
      A1(2,3)= (ZQ+MASS)*U0+2.0*CDZ*AREA*XA*DABS(W0)
      A1(2,4)= -(WEIGHT-BUOY)*DSIN(THETA0)

C      A1(3,1)= MW*W0+2.0*U0*(MDS*DS+MDB*DB)
      A1(3,2)= MW*U0+2.0*CDZ*AREA*XA*DABS(W0)
      A1(3,3)= (MQ-MASS*XG)*U0-MASS*ZG*W0
      &      -2.0*CDZ*IA*DABS(W0)
      &      A1(3,4)= (XG*WEIGHT-XB*BUOY)*DSIN(THETA0)-
      &      (ZG*WEIGHT-ZB*BUOY)*DCOS(THETA0)

C      A1(4,1)= 0.0
      A1(4,2)= 0.0
      A1(4,3)= 1.0
      A1(4,4)= 0.0

C      A2(1,1)= KP*U0+(KWP-MASS*ZG)*W0
      A2(1,2)= -(ZG*WEIGHT-ZB*BUOY)*DCOS(THETA0)
      A2(1,3)= KV*U0+KVW*W0
      A2(1,4)= (KR+MASS*ZG)*U0+KWR*W0

C      A2(2,1)= 1.0
      A2(2,2)= 0.0
      A2(2,3)= 0.0
      A2(2,4)= DTAN(THETA0)

C      A2(3,1)= YP*U0+(YWP+MASS)*W0
      A2(3,2)= (WEIGHT-BUOY)*DCOS(THETA0)
      A2(3,3)= YV*U0+YVW*W0-CDZ*AREA*DABS(W0)
      A2(3,4)= YWR*W0+(YR-MASS)*U0-CDZ*AREA*XA
      &      *DABS(W0)

C      A2(4,1)= MASS*XG*W0+NP*U0+NWP*W0
      A2(4,2)= (XG*WEIGHT-XB*BUOY)*DCOS(THETA0)
      A2(4,3)= NV*U0+NVW*W0-CDZ*AREA*XA*DABS(W0)
      A2(4,4)= (NR-MASS*XG)*U0+NWR*W0-CDZ*IA*DABS(W0)

C      RESTORE B-MATRIX
C
C      DO 71 I=1,4
      DO 72 J=1,4
      BB1(I,J)=B1(I,J)
72      CONTINUE
71      CONTINUE

C      DO 81 I=1,4
      DO 82 J=1,4
      BB2(I,J)=B2(I,J)
82      CONTINUE
81      CONTINUE

```

```

C      CALL RGG(4,4,A1,BB1,ALFR1,ALFI1,BETA1,0,ZZZ1,IER)
      CALL DEGSTB(DEOS1,ALFR1,ALFI1,BETA1,FREQ1,WR1,WI1)

C      CALL RGG(4,4,A2,BB2,ALFR2,ALFI2,BETA2,0,ZZZ2,IER)
      CALL DEGSTB(DEOS2,ALFR2,ALFI2,BETA2,FREQ2,WR2,WI2)

C      IF (DEOS1.GE.DEOS2) DEOS=DEOS1
      IF (DEOS1.LT.DEOS2) DEOS=DEOS2

C      WRITE (41,2001) DSD,THET0,U0,W0,WP,DEOS,
&      DEOS1,DEOS2
      IF (DEOS.LT.0.D0)
&      WRITE (42,2001) DSD,THET0,U0,W0,WP,DEOS,
&      DEOS1,DEOS2
      IF (DEOS1.LT.0.D0)
&      WRITE (43,2001) DSD,THET0,U0,W0,WP,DEOS,
&      DEOS1,DEOS2

C      IF (IRES.EQ.1) GO TO 11
      IF (IRES.EQ.2) GO TO 12
      IF (IRES.EQ.3) GO TO 13
      IF (IRES.EQ.4) GO TO 14

C      100 STOP
      1001 FORMAT (' ENTER THE RESPONSE DATA FILE DESIRED
&      (1,2,3, OR 4) ')
      2001 FORMAT (8E15.5)
      2002 FORMAT (F10.3)
      END

C      SUBROUTINE DEGSTB(DEOS,ALFR,ALFI,BETA,OMEGA,WR,WI)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION ALFR(4),ALFI(4),BETA(4),WR(4),WI(4)
      DO 1 I=1,4
        WR(I)=ALFR(I)/BETA(I)
        WI(I)=ALFI(I)/BETA(I)
1    CONTINUE
      DEOS=-1.0E+10
      DO 2 I=1,4
        IF (WR(I).LT.DEOS) GO TO 2
        DEOS=WR(I)
        IJ=I
2    CONTINUE
      OMEGA=WI(IJ)
      OMEGA=DABS(OMEGA)
      RETURN
      END

C      SUBROUTINE TRAP(N,A,B,OUT)

```

C
C
C
C

NUMERICAL INTEGRATION ROUTINE USING
THE TRAPEZOIDAL RULE

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION A(1),B(1)
N1=N-1
OUT=0.0
DO 1 I=1,N1
 OUT1=0.5*(A(I)+A(I+1))*(B(I+1)-B(I))
 OUT =OUT+OUT1
1 CONTINUE
RETURN
END

LIST OF REFERENCES

1. Smith, N.S., Crane, J.W., and Summey, D.C., "SDV Simulator Hydrodynamic Coefficients," NCSC Technical Memorandum 231-78, June 1978.
2. Booth, T.B., "Stability of Buoyant Underwater Vehicles, Predominantly Forward Motion," **International Shipbuilding Progress**, Volume 24, November 1977, Nr. 279, pages 297-305.
3. Booth, T.B., "Stability of Buoyant Underwater Vehicles, Near Vertical Ascent," **International Shipbuilding Progress**, Volume 24, December 1977, Nr. 280, pages 346-352.

BIBLIOGRAPHY

Boncal, R.J., "A Study of Model Based Manuevering Controls for Autonomous Underwater Vehicles," Master's Thesis, Naval Postgraduate School, Monterey, California, December 1987

Gueler, G.F., "Modelling, Design, and Analysis of an Autopilot for Submersible Vehicles," **International Shipbuilding Progress**, Volume 36, February 1989, Nr. 405, pages 51-85.

Richards, R.J. and Stoten, D.P., "Depth Control of a Submersible Vehicle," **International Shipbuilding Progress**, Volume 28, February 1981, Nr. 318, pages 30-39.

INITIAL DISTRIBUTION LIST

| | | No. Copies |
|----|---|------------|
| 1. | Defense Technical Information Center Cameron Station Alexandria, Virginia 22304-6145 | 2 |
| 2. | Library, Code 52 Naval Postgraduate School Monterey, California 93943-5002 | 2 |
| 3. | Department Chairman, Code ME Department of Mechanical Engineering Naval Postgraduate School Monterey, California 93943-5000 | 1 |
| 4. | Professor Fotis A. Papoulas Code ME/Pa Department of Mechanical Engineering Naval Postgraduate School Monterey, California 93943-5000 | 2 |
| 5. | LT Brian D. McKinley Navy Experimental Diving Unit Panama City, Florida 32407 | 2 |
| 6. | Naval Engineering Curricular Officer, Code 34 Department of Mechanical Engineering Naval Postgraduate School Monterey, California 93943-5004 | 1 |